Quiz # 7 Solutions

1. \[2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0\]
so \[\frac{dx}{dt} = -\frac{y \cdot \frac{dy}{dt}}{x} = -\frac{4 \cdot 6}{3} = -8\]
because if \(y = 4\) then \(x = 3\) so \(x^3 + y^2 = 25\).

2. The surface area is a function of the radius, and the diameter is a function of the radius:
\[A = 4\pi r^2\]
and \(D = 2r\).
Thus \[A = 4\pi r^2 = \pi (2r)^2 = \pi D^2\]
and \[\frac{dA}{dt} = 2\pi D \frac{dD}{dt}\].
We want \(dD/dt\):
\[\frac{dD}{dt} = \frac{dA}{2\pi D} = -\frac{1}{2\pi \cdot 10} = -\frac{1}{20\pi} \approx -1/60 \text{ cm/min.}\]
That is, the diameter decreases at \(1/(20\pi)\) cm/min.

3. by LINEARIZATION:
\(f(x) = x^4\) and \(a = 2\) gives
\[L(x) = f(a) + f'(a)(x - a) = 16 + 4 \cdot 8(x - 2) = 16 + 32(x - 2)\]
because \(f'(x) = 4x^3\). Thus
\[(2.01)^4 = f(2.01) \approx L(2.01) = 16 + 32(2.01 - 2) = 16 + 32(0.01) = 16.32.\]

by DIFFERENTIALS:
\(f(x) = x^4\) so
\[dy = f'(x) \, dx = 4x^3 \, dx\]
Here \(x = 2\) and \(dx = .01\) so
\[dy = 4 \cdot 8 \cdot .01 = .32\]
and
\[(2.01)^4 = 2^4 + \Delta y \approx 2^4 + dy = 16 + .32 = 16.32.\]
[Note that \((2.01)^4 = 16.32240801\) exactly.]

4.
\[A: \text{ local maximum OR neither}\]
\[B: \text{ neither}\]
\[C: \text{ local maximum}\]
\[D: \text{ absolute maximum}\]
\[E: \text{ local minimum}\]

[On point “A” the book’s definition of “local maximum/minimum” implies that the correct answer is neither, but based on what I had said in class you would be correct to write local maximum. I will accept either answer at endpoints, in the future.]