Quiz # 5 Solutions

1. Prove that \( \frac{d}{dx} (\sec x) = \sec x \tan x \).

Solution:
\[
\frac{d}{dx} (\sec x) = \frac{d}{dx} \left( \frac{1}{\cos x} \right) = \frac{0 \cdot \cos x - 1 \cdot (-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos x \cdot \cos x} = \sec x \tan x.
\]

2. (a) Find the derivative: \( f(x) = \sec x / (x - \cot x) \).

Solution:
\[
f'(x) = \frac{(\sec x \tan x)(x - \cot x) - (\sec x)(1 + \csc^2 x)}{(x - \cot x)^2}.
\]
[There is no need to simplify, though you may.]

(b) Find the derivative: \( y = \cos(e^x) \).

Solution:
\[
\frac{dy}{dx} = -\sin(e^x) \cdot e^x.
\]

3. The position function of a particle is given by \( s = t^3 - 4.5t^2 - 7t \), where \( s \) is measured in meters and \( t \) is the (positive) time measured in seconds. When does the particle reach a velocity of 5 m/s?

Solution:
\[
\dot{s} = 3t^2 - 9t - 7.
\]
The velocity is \( \dot{s} \). When is \( \dot{s} = 5 \)?:
\[
3t^2 - 9t - 7 = 5
\]
\[
3t^2 - 9t - 12 = 0
\]
\[
t^2 - 3t - 4 = 0
\]
\[
(t + 1)(t - 4) = 0
\]
So \( t = -1 \) or \( t = 4 \). But we want the positive time only. Thus the answer is \( t = 4 \) seconds.

4. Find the points on the graph of \( y = x + 2 \cos x \) where the tangent line is horizontal.

Solution:
\[
\frac{dy}{dx} = 1 + 2(-\sin x) = 0
\]
\[
\sin x = \frac{1}{2}
\]
so
\[
x = \frac{\pi}{6} + 2k\pi \quad \text{or} \quad x = \frac{5\pi}{6} + 2k\pi
\]

for some integer \( k \).