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Summary of current research and future research plans

My research interests include nonharmonic Fourier series, control theory of distributed parameter systems and inverse problems of mathematical physics (including control and inverse problems on graphs), signal processing, partial differential and differential-difference equations, and mathematical modeling of nanodevices.

A significant part of my current research interests are focused on the development of methods and algorithms for solving control and identification problems in distributed systems. My approach is based on deep and heretofore incompletely exploited connections between nonharmonic Fourier series, control theory for partial differential equations, inverse problems of mathematical physics, and signal processing. These connections can be summarized as follows:

On the one hand, the method of moments connects properties of exponential families with control problems for partial differential equations. The recently developed effective Boundary Control (BC) method in inverse problems is based on connections between controllability and identification problems for systems described by partial differential equations. On the other hand, sampling theory can be formalized in terms of Riesz bases of exponentials in certain L^2 spaces. Application of Boundary Control ideas leads to a determination of such bases for unions of intervals, thus giving a generalization of the Shannon–Kotel'nikov theorem for signals with multi-band spectra. Another significant problem of signal processing – spectral estimation – turns out (from the viewpoint of system theory) to be closely connected with inverse problems.

More specifically, my current research and near-future research plans include:

Inverse problems in glaciology. The basal boundary of glaciers and ice sheets is generally not observationally accessible. This poses a major problem for glacier modeling studies, because the basal boundary condition is an essential part of a well-posed problem. The surface, however, is accessible to ground based, airborne, and satellite measurements. It is now possible to measure surface topography, ice thickness, and surface velocities over large areas. This sets up a classic inverse problem, with too many boundary conditions at the top and not enough at the bottom. The application of inverse methods will lead to derived basal velocity fields, also over large areas. This will be a major improvement in the understanding of the subglacial environment, and it will represent a major step towards predictive glacier models. Such models are important in the study of glacier-climate interaction, dating of ice cores, and assessment of natural hazards.

We propose several approaches to this very complicated inverse problem, including special iterative procedures. Our proposals resulted in the award of two NSF grants ($360 K and $400 K).

Control and inverse problems for partial differential equations on graphs. Differential equations on graphs are used to describe many physical processes such as mechanical vibrations of multi-linked flexible structures (usually composed of flexible beams or strings), propagation of electro-magnetic waves in networks of optical fibers, heat flow in multi-link networks, and also electron flow in quantum mechanical circuits. Recently mesoscopic quasi-one-dimensional structures (graphs), like quantum, atomic, and molecular wires, have become the subject of extensive experimental and theoretical studies. According to physical terminology, a quantum wire is a graph-like structure on the surface of a semiconductor, which confines an electron to potential grooves of a width of a few nanometers. The simplest model describing conduction in quantum wires is a Hamiltonian on a planar graph. A solid theory of the inverse problems for the Schrödinger equation on graphs would be an important step towards designing quantum devices. Unfortunately, to date there are only a few results concerning inverse problems on graphs. We plan to develop a new, effective, and robust approach for solving the inverse problems for partial differential equations on graphs. Our approach will be based on the Boundary Control method and will use the results on controllability of multi-link flexible structures. It will be applied to graphs with cycles and to graphs with control/observation on a small part of the boundary when only spectral controllability may happen.

Control and identification problems for systems with internal damping and for hybrid systems. We plan to generalize the Boundary Control method in inverse problems for damped and hybrid systems. This will have important applications in many areas, such as material science, nondestructive testing, geophysics, acoustic imaging, and remote sensing.

Recently, control theory specialists have shown considerable interest in modelling, control, and identification problems for constrained layer structures. In particular, in the book "Smart Material Structures. Modelling, Estimation and Control", by H.T. Banks et al., parameter estimates are obtained for a cantilevered beam with piezoceramic patches. Material parameters, such as Young's modulus and sensor constants related to the piezoceramic patches, are estimated from dynamic observations of the governing system. These observations (beam acceleration, velocity data, displacement data taken at various locations of the beam depending on the measuring devices at hand) are then used in an optimization problem, where a least squares output functional of the parameters in question is considered.

We will demonstrate, in particular, the possibility of determining the location of the piezoceramic patches, as well as the level of voltages in them, using boundary observation. Our approach uses a modification of the BC method; it does not involve nonlinear optimization procedures. We obtained also new results on controllability of the wave and beam equations with structural damping.

Control and identification problems for the Schrödinger equation. Control and inverse problems for quantum mechanical systems are important for new applications such as quantum computers and nanotechnology. We consider the inverse problem of determining the potential in the Schrödinger equation from dynamical boundary observations. Dynamical boundary data have not been used in the inverse problem for the Schrödinger equation, since the traditional Gelfand–Levitan–Marchenko (GLM) approaches reconstruct the potential from scattering or spectral data. We show that one can completely recover the spectral data from the dynamical data, using properties of exact and spectral controllability for the Schrödinger equation. We extend the BC method for these problems, and we use propagation of singularities for the wave equation. Our approach is essentially dimension-independent, and it lends itself to straightforward algorithmic implementations and stable numerical schemes. This is important, since GLM approaches work only in one spatial dimension, while the reconstruction method proposed by Faddeev for general multidimensional problems is extremely complicated and in practice is not amenable to reliable and robust numerical algorithms.

We have also developed the Boundary Control approach to the Titchmarsh-Weyl function recovering new connections of our method with the method of B. Simon.

Construction of sampling and interpolating sets for signals with multi-band spectra. This problem is important for signal processing and has numerous applications in physics, engineering, and defense. Its solution yields both stable and non–redundant sampling of multi-band signals, and it gives a generalization of the Whittaker–Kotel'nikov–Shannon sampling formula, which has fundamental significance for accurate and robust transmission of information.

This formula works when the spectrum of a signal is a finite interval of the real axis. Very little is known about sampling and interpolating sequences in the case of several intervals. The reason is that, although the concepts of sampling and interpolating are within classical function theory, none of the well-known tools developed for such problems (infinite products, interpolation series, etc.) work in this situation.

We have proposed several new approaches to the problem, which are of significant interest in their own right. Our first approach is based on connections between controllability of dynamical systems described by linear partial differential equations, problem of moments, and the Riesz basis property of corresponding exponential families. We have related the problem of constructing the sampling set for spectrally constrained signals to the solution of certain kinds of Wiener-Hopf equations, and we have described new techniques for solving these equations. This research has been supported by an Australian Research Council grant.

The ultimate aim of the work is to obtain a description, for multi-band signals, of all sampling and interpolating sequences.

Developing of an exact effective method in the spectral estimation problem, i.e. recovery of unknown N, a_n, and λ_n in a signal $f(t) = \sum_{n=1}^{N} a_n e^{\lambda_n t}$ by given samples $f(t_j), j = 1, \ldots, 2N$. There is an enormous literature devoted to this important problem in signal processing. We propose a new method based on our approach to dynamical inverse problems and their connections with control-theoretic ideas. This approach yields simple, fully linear recovery algorithms for the unknown parameters. The new method allows us to study various generalization of the spectral estimation problem, including cases of multiple spectra, irregular sampling, and signals in the presence of noise.

Mathematical modeling of nanodevices. In the framework of the DOD supported "Spintronics" project we have developed a mathematical model of a spin polarizing device (spin gun). For this purpose we studied the problem of spin-dependent transport of electrons through a finite array of quantum dots attached to a 1D quantum wire, for various semiconductor materials. The Breit-Fermi term for spin-spin interaction in the effective Hamiltonian of the device is shown to result in a dependence of the transmission coefficient on the spin orientation. The difference in transmission probabilities for singlet and triplet channels can reach a few percent for a single quantum dot. For several quantum dots in an array, due to interference effects, the difference can reach approximately 100% for some energy intervals. For these same energy intervals the conductance of the device reaches a value of approximately unity in $[e^2/\pi\hbar]$ units. As a result, a model of a spin-gun which transforms a spin-unpolarized electron beam into an approximately 100% polarized beam has been suggested.

Now we are working on the problem of incorporation an internal structure of the quantum dots into our model. This more complicated model is important and interesting for the following reasons: (i) The presence of internal energy levels of quantum dots leads to a reach structure of resonances in the external channel. As a consequence there is a possibility for better controllability of the conductance and the polarization efficiency of the device. (ii) Taking account of the internal structure of quantum dots will also extend the possible choices of semiconductor materials.