Recursion vs. Iteration

CS 311 Data Structures and Algorithms
Lecture Slides
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Glenn G. Chappell
Department of Computer Science
University of Alaska Fairbanks
ggchappell@alaska.edu

Some material contributed by Chris Hartman

Unit Overview
Recursion & Searching

Major Topics

- ✓ Introduction to recursion
- ✓ Search algorithms I
- Recursion vs. iteration
- Search algorithms II
- Eliminating recursion
- Search in the C++ STL
- Recursive backtracking

Review

Review
Search Algorithms I — Binary Search

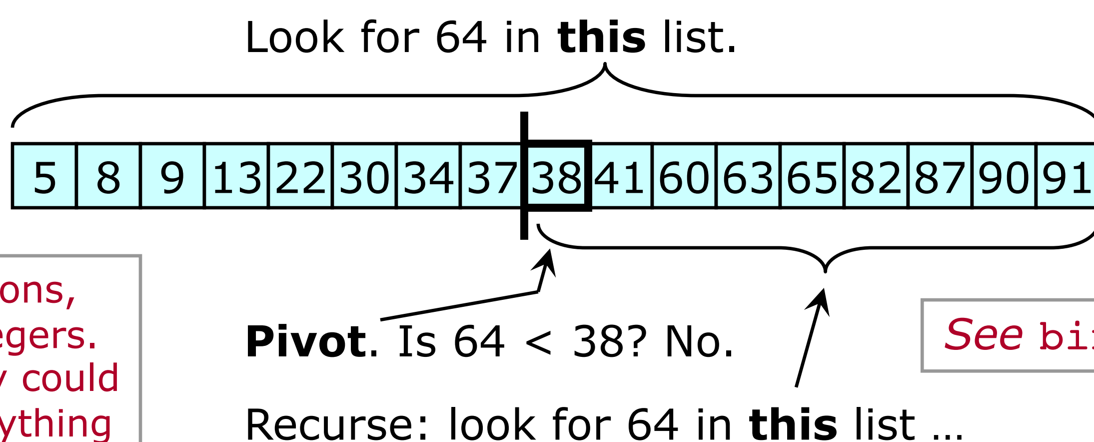
The **Binary Search** algorithm finds a given **key** in a **sorted list**.

- Here, *key* = thing to search for. Often there is associated data.
- In computing, **sorted** means in (some specified) order.

Procedure

- Pick an item in the middle of the list: the **pivot**.
- Compare the given key with the pivot.
- Using this, narrow search to top or bottom half of list. Recurse.

Example: Use Binary Search to search for 64 in the following list.



In my illustrations, keys will be integers. In practice, a key could be just about anything that can be sorted.

See `binsearch1.cpp`.

Review
Search Algorithms I — Better Binary Search

Equality vs. Equivalence—may not be the same when objects being compared are not numbers.

- **Equality**: `a == b`.
- **Equivalence**: `!(a < b) && !(b < a)`.

Using equivalence instead of equality in Binary Search:

- Maintains consistency: always compare with `operator<`.
- Allows use with value types that do not have `operator==`.

See `binsearch2.cpp`*.*

Using Operators Random-access iterators only	**Using STL Function Templates** Works with all forward iterators Still fast with random-access
`iter += n`	`std::advance(iter, n)`
`iter2 - iter1`	`std::distance(iter1, iter2)`

Recursion vs. Iteration

Recursion vs. Iteration
Fibonacci Again — Faster

We wrote a function that, given *n*, returns Fibonacci number *n*. For *n* > 40, our function is extremely slow.

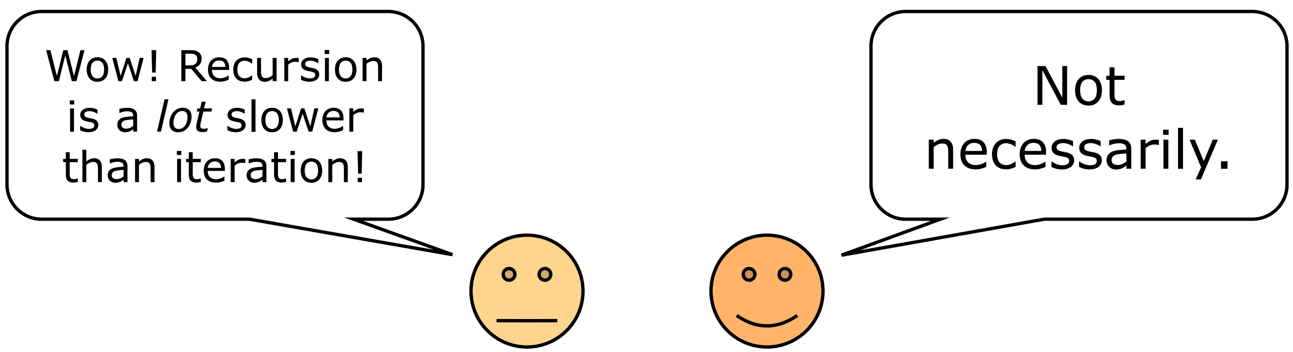
See `fibo_first.cpp`*.*

What can we do about this?

TO DO

- Rewrite the Fibonacci computation in a fast **iterative** form (using loops).

Done. See `fibo_iterate.cpp`*.*



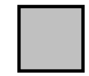
TO DO

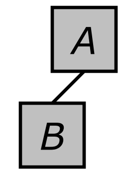
- Figure out how to do a fast *recursive* Fibonacci computation. Write it.

Done. See `fibo_recurse.cpp`*.*

Recursion vs. Iteration
Fibonacci Again — Note on Trees

Use a **tree** to represent function calls some algorithm makes.

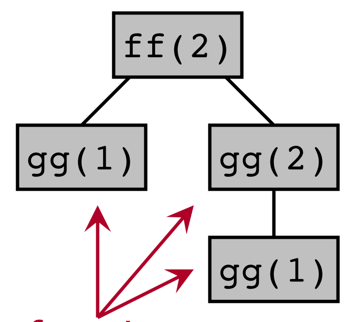
- A box represents making a call to a function. 
- A line from an *A* box down to a *B* box represents this call to function *A* making a call to function *B*.




```
int ff(int n)
{
    return gg(n-1) + gg(n);
}

int gg(int k)
{
    if (k <= 1) return 7;
    else        return 2*gg(k-1);
}
```

Tree representing calls made by doing `ff(2)`



Same function.
Different **invocations**
of that function.

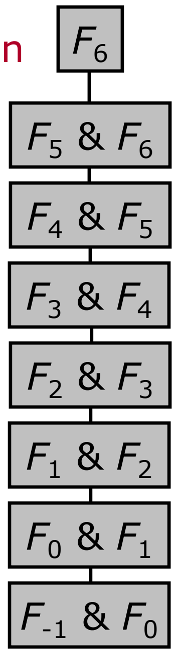
Fibonacci Again — Comments [1/3]

Choice of algorithm can make a *huge* difference in performance.

Computing F_6

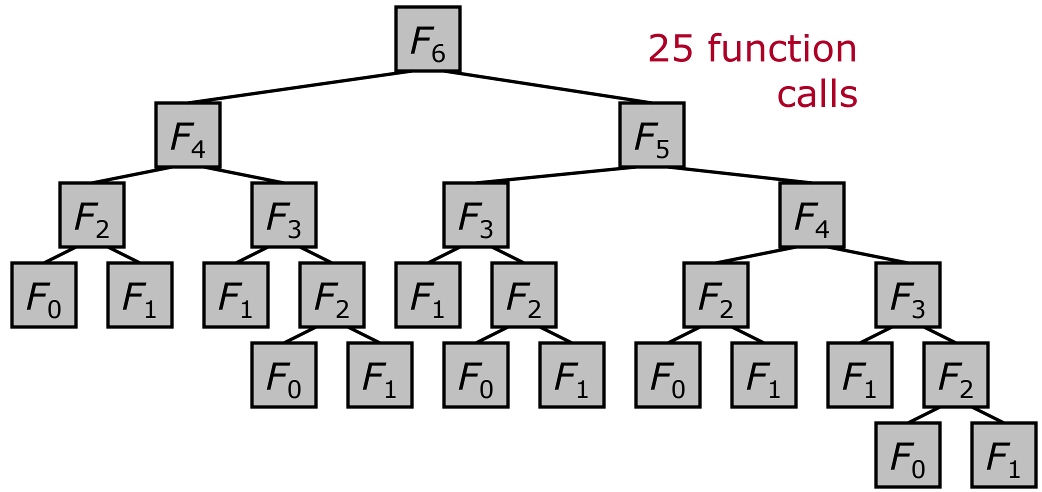
`fibo_recurse.cpp`

8 function calls



`fibo_first.cpp`

25 function calls



Fibonacci No.	`fibo_recurse.cpp`	`fibo_first.cpp`
F_7	9 calls	41 calls
F_{10}	12 calls	177 calls
F_{20}	22 calls	21,891 calls
F_{40}	42 calls	331,160,281 calls

Recursion vs. Iteration
Fibonacci Again — Comments [2/3]

A `struct` can be used to return two values at once.

- Templates `std::pair` (`<utility>`) and `std::tuple` (`<tuple>`) can be helpful.

The 2017 C++ Standard introduced **structured bindings**, making this more convenient.

```
tuple<bignum, bignum> fibo_recurse(int n);

auto [a, b] = fibo_recurse(k);
```

Now `a` and `b` are variables of type `bignum`.

`a` is `fibo(k-1)`.

`b` is `fibo(k)`.

Recursion vs. Iteration
Fibonacci Again — Comments [3/3]

Some algorithms have natural implementations in both **recursive** and **iterative** form.

Sometimes we have a **workhorse** function that does most of the processing, and a **wrapper** function with a convenient interface.

- Often the wrapper just calls the workhorse for us.
- This is common when we use recursion, since recursion can place restrictions on how a function is called.

We have seen this idea in another context. Recall `toString` and `operator<<` from Project 1.

```
cout << p.toString();
```

If we had not written our own `operator<<`, then we could still do this.

```
cout << p;
```

With our `operator<<`, we can do this. So `operator<<` is really just a convenient wrapper around `toString`.

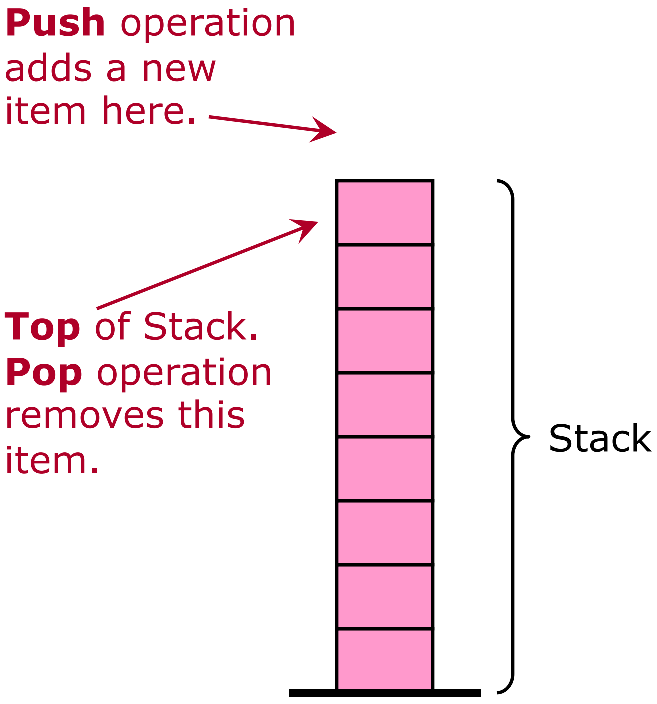
Recursion vs. Iteration
Function-Call Internals [1/4]

To fully grasp the issues involved in recursion vs. iteration, it helps to understand how function calls are implemented.

A running program makes use of a structure called the **call stack** (there are other names, all involving the word "stack").

A Stack is a kind of container. We will look at Stacks in detail later in the semester. For now:

- Think of a stack of plates. We can place a new plate on top, and we can pull a plate off the top. We only deal with the **top** of the Stack.
- Adding something on top is a **push** operation. Taking something off the top is a **pop** operation.



Recursion vs. Iteration
Function-Call Internals [2/4]

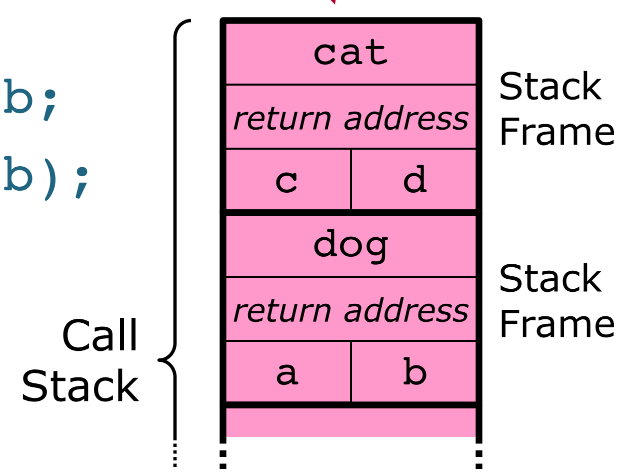
The items on the call stack are **stack frames**. Each stack frame corresponds to an **invocation** of a function.

- A function's stack frame holds:
  - Its automatic variables, including parameters.
  - Its **return address**: where to go back to when it returns.
- When a function is called, a stack frame for that function is pushed.
- When the function exits, its stack frame is popped.

```
void cat(Foo c)
{
    int d;
    llama();
    …
}
```

At **this** point in the code (assuming `cat` was called by `dog`), the call stack looks like **this**.

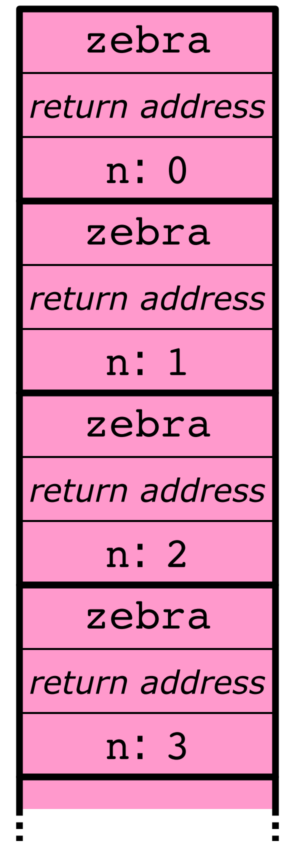
```
void dog(int a)
{
    Foo b;
    cat(b);
}
```



Recursion vs. Iteration
Function-Call Internals [3/4]

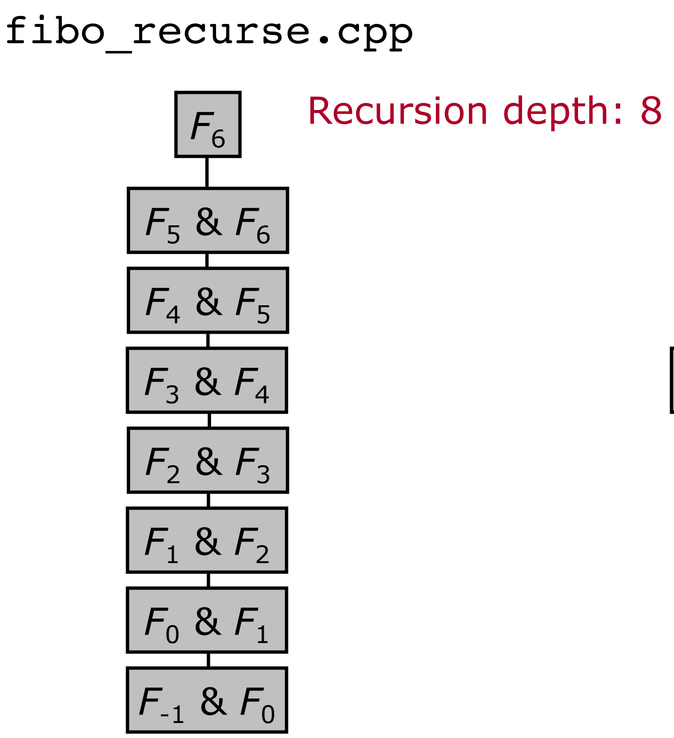
When a function calls itself recursively, there will be multiple stack frames on the call stack corresponding to the *same* function—but *different invocations* of that function.

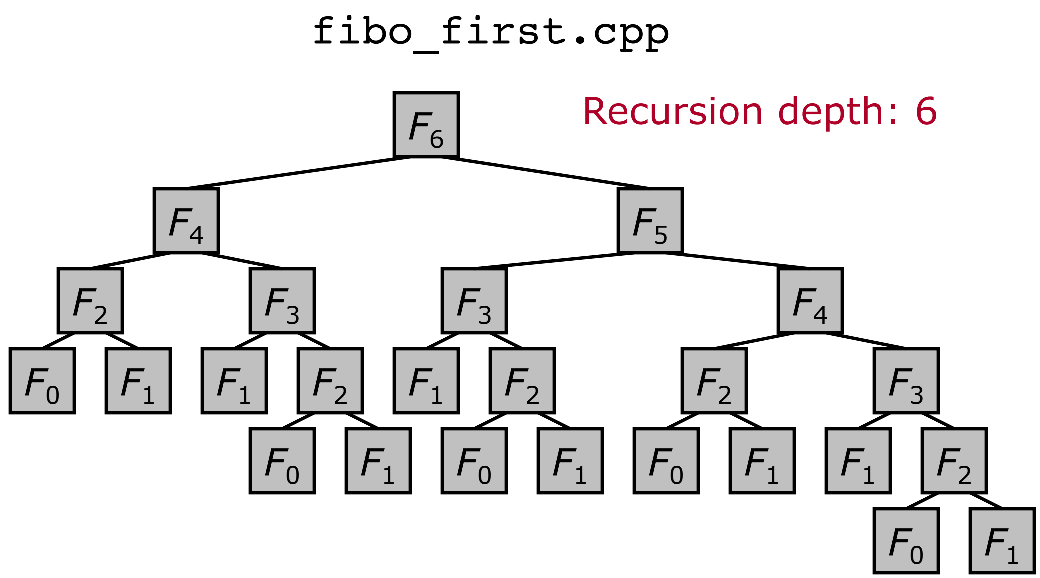
```
void zebra(int n)
{
    if (n == 0)
    {
        cout << n << endl;
        return;
    }
    cout << n << " ";
    zebra(n-1);
}
```



Recursion vs. Iteration
Function-Call Internals [4/4]

A function call's **recursion depth** is the greatest number of stack frames on the call stack *at any one time* as a result of the call.





When analyzing *time* usage, the total number of calls is of interest.

When analyzing *space* usage, the recursion depth is of interest.

Recursion vs. Iteration
Drawbacks of Recursion

Two factors can make recursive algorithms inefficient.

- Inherent inefficiency of some recursive algorithms
  - But there are efficient recursive algorithms.
- Function-call overhead
  - Making all those function calls requires work: pushing and popping stack frames, saving return addresses, creating and destroying automatic variables.

These two are important regardless of the recursive algorithm used.

And recursion has another problem.

- Memory-management issues
  - A high recursion depth causes the system to run out of memory for the call stack. This is **stack overflow**, and it generally cannot be dealt with using normal error-handling procedures. The result is usually a crash.
  - When we use iteration, we can manage memory ourselves. This can be more work for the programmer, but it also allows proper error handling.

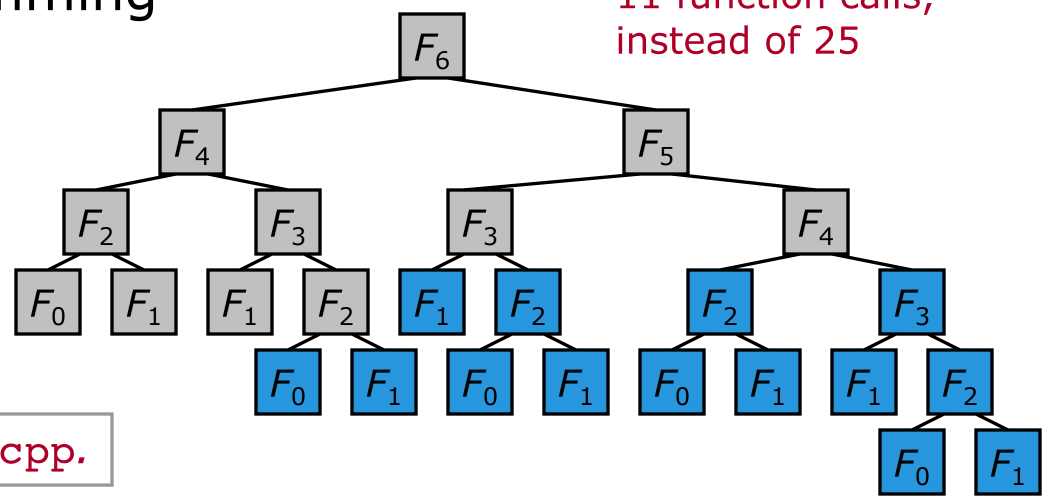
Recursion vs. Iteration
Fibonacci Yet Again — Dynamic Programming

Dynamic programming (which does *not* mean what it sounds like) can greatly speed up some recursive algorithms.

- It involves saving the results of recursive calls so that they do not need to be recomputed.
- In some contexts, this technique is called **memoizing**.
- Dynamic programming is covered in CS 411.

If we apply dynamic programming to `fibo_first.cpp`, then the recursive calls shown in **blue** no longer need to be made.

11 function calls, instead of 25



See `fibo_dp_topdown.cpp`*.*

Recursion vs. Iteration
Fibonacci Yet Again — Formula

There is a simple formula for F_n, using non-integer computations.

Let $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.6180339$. (This is often called the **golden ratio**.)

For each nonnegative integer n, F_n is the nearest integer to $\frac{\varphi^n}{\sqrt{5}}$.

Here is `fibo` using this formula:

A floating-point literal with an "`L`" added at the end is of type `long double`.

```
bignum fibo(int n)
{
    long double phi = (1.0L + sqrt(5.0L)) / 2.0L;
    long double near_fibo = pow(phi, n) / sqrt(5.0L);
    // Our Fibonacci number is the nearest integer
    return bignum(near_fibo + 0.5L);
}
```

See `fibo_formula.cpp`*.*

Recursion vs. Iteration
Fibonacci Yet Again — Very Fast

An even faster method of computing Fibonacci numbers relies on the following facts:

- $F_{2n-1} = (F_{n-1})^2 + (F_n)^2$.
- $F_{2n} = 2F_{n-1}F_n + (F_n)^2$.

For the fast methods we mentioned earlier, computing F_n requires something like *n* arithmetic operations. But using the above facts, we can compute F_n using something like log *n* arithmetic operations—much less, when *n* is large.

This allows for easy computation of Fibonacci numbers that are much larger than any C++ built-in integer type can hold. To illustrate the power of this method, I have implemented it in Python, which has a built-in arbitrarily large integer type.

See `fibo_fast.py`*.*

Recursion vs. Iteration
Fibonacci Yet Again — Comments

A single problem may be solvable by many different methods.

- Different methods can have very different performance characteristics.
- It is possible that a very efficient method is not at all obvious.

Computing Fibonacci numbers is not something we need to do very often, in practice. But the above observations apply to other problems as well.

Next we will return to the problem of finding a key in a list.