Introduction

Service in the airline industry has made headlines for years, but rarely due to its excellence. Traveling by air has traditionally been one of the more stressful modes of transit, with long periods spent waiting, little personal space, mandatory security checks, and the odd erratic scheduling change with little to no advance warning. Several pieces of legislation have been passed with the aim of making air travel more comfortable for the average passenger, but for a specific group of people--those with limited mobility--their ability to fly is still dependent on the availability of a rolling seat, which can make the experience of flying with a commercial carrier especially trying.

Regulations in the industry have made things a little bit easier. Large modern airplanes have to be built in such a way that passengers requiring the use of a wheelchair are able to roll to their seats and use the facilities, along with allowing in-cabin space for one folded personal wheelchair. The airports, however, cause their own sets of problems. Treks between gates are longer than much of the population can walk without requiring assistance, and with the enactment of the Air Carrier Access Act (US Department of Transportation, 2003), it has become the responsibility of the airlines to ensure that these people are able to get to their gate, make their connections, and board their planes without suffering undue inconvenience or embarrassment. The subject of this paper is how to do exactly that while minimizing costs to the airline and subsequently the consumer.

What Epsilon and all other airlines are required by law to provide is clearly defined. They are not to require advance notification if a passenger will be using a wheelchair. It is the sole responsibility of the carrier to provide wheelchairs, wheelchair ramps and lifts, and personnel to assist where needed to make sure that the passengers can enplane, deplane, make connections, and travel between gates. A traveler who is not independently mobile is not to be left without an attendant for more than 30 minutes. Carriers are not permitted to restrict the movement of disabled passengers to a holding area or other special location.

As far as training is concerned, the airline is required to employ staff versed in the specifics of pertinent regulations and the available equipment, as well as how to respectfully deal with people with disabilities. The airline is not, however, required to train staff in medical services, and the staff is not required to help persons with eating or facilitation of toilet functions, nor should they have to lift them at any time. Written complaints by persons with disabilities are also to be catalogued and a report submitted annually (US Department of Transportation, 2003). This leaves the main duties of the carrier providing wheelchairs and assistance in traversing the airport with them in the form of escorts, in addition to providing staffing to deal with complaints.

Graph Traversal Simulation Model

Overview:

The graph traversal simulation model describes an airport as a weighted, directed graph. The strengths and weaknesses of a wheelchair distribution method are determined through a Monte Carlo simulation of the workings of the airport. Passengers are modeled as individual entities, each with a specific flight to catch, and disabled individuals are assisted by employees designated as escorts and provided wheelchairs from the airline's wheelchair caches.

Each graph node is associated with part of the airport. The node is given a distance passengers must traverse in order to reach adjoining nodes, and also designated as one of the following types:

- start node, indicating an exit to the street
- wheelchair cache, for wheelchair storage
- check-in area, where passengers wait in line to check in
- baggage claim, where deplaned passengers pick up their baggage before exiting
- a free area, for travel between other nodes

To travel to an adjacent node, a passenger or employee must traverse the length of their current node, given by its size. Each person has a base walk speed, determined randomly from the distribution shown in Assumptions Figure 1, and traverse the node at a speed given by:

$$walk\ speed = base\ speed * congestion\ factor$$

where

$$congestion\ factor = \#\ of\ people\ in\ node\ /\ max\ population\ of\ node$$

The max population of a node is determined by allowing one person per square meter.

Passengers use Dijkstra's shortest path algorithm(see Appendix Dijkstra) to determine the list of nodes they must visit in order to reach their destination. Each edge is given an initial cost of 1. If the passenger has passed through security, edges leading out of the security area are given a cost of 100. This gives passengers a strong incentive not to leave a security area once they have entered it.

Passenger Behavior:

Passengers enter into the simulation either from the street or when deplaned from a new arrival. Passengers appear from the street t minutes prior to their flight, where t is given by the following distribution:

The behavior of the passengers is described by the following pseudo-code:

```
if enteredFromStreet
      take shortest path to check-in counter
      if disabled
            call for escort
            wait for escort
            //  if the passenger does not need to be escorted to their
destination...
            if onlyNeedWheelchair
                  take wheelchair
                  dismiss escort
      go to gate
      if beingEscorted
            dismiss escort
      wait for flight
      if hasWheelchair
            leaveWheelchair
      leave simulation
else if arrived
      if disabled
            call for escort
            wait for escort
            if onlyNeedWheelchair
                  take wheelchair
                  dismiss escort
      if hasConnection
            go to gate
            if beingEscorted
                  dismiss escort
            wait for flight
      else
            go to baggage
            //  passenger picks up their own wheelchair
            if beingEscorted
                  dismissEscort
            take shortest path to exit
      if hasWheelchair
            leaveWheelchair
      leave simulation
```

Escort Behavior:

Escorts enter the building via the parking lot at the beginning of the simulation. They then begin executing this loop:

```
while simulation is running
      if there is a passenger that needs assistance
            remove wheelchair from cache
            take shortest path to passenger
            if passenger needs escort
                  take passenger to destination
                  give passenger control of wheelchair
                  return to nearest wheelchair cache
            else
                  give passenger wheelchair
                  return to nearest wheelchair cache
      else
            if there is a dolly available
                  collect all wheelchairs left around airport
                  return to nearest wheelchair cache
```

Assumptions of Simulation:

An airport is a complex system whose workings involve an enormous number of physical and psychological factors, as well as outside influences. While our simulation captures many of these, we have had to make many assumptions and simplifications.

Airport Node Representation:
- The node representation for travel distances are assumed to be adequate representations of the true airport layout.
- Passengers who miss flights leave the airport.
- Delays between flights are scheduled randomly as described in Appendix FOO.
- Connecting flights for deplaning passengers are chosen randomly from the next 5 flights.
- Disabled passengers who need assistance once they are at the gate get help from free flight attendants rather than call an escort.
- Check-in and security points have parameter for number of lines each with a queue of passengers.

Passenger Assumptions:
- Passengers travel directly to their gate via the shortest path
- Once at their gate, passengers do not move until their flight takes off.
- Passenger arrival at check-in is described by the distribution shown in Figure 1.
- Passenger service times at check-in described by a distribution with minimum of 2 minutes, maximum of 20 minutes, and peak at 5 minutes

- Passenger service times at security are described by the same distribution with a peak at 3 minutes.
- Passenger walk speeds are described by the distribution in Figure 2.
- Passengers in wheelchairs, either self-propelled or pushed by an escort, have base speeds described by the same distribution as other passengers (see Figure 2).
- Movement speed through a node is a function of base walk speed and density of individuals in that node.
- Passengers deplane and enplane simultaneously and instantly.
- Passengers are extremely unlikely to leave a security zone once they have entered it.
- Passengers who do not arrive prior to the required 30 minute check-in time leave.
- Disabled passengers will lodge a complaint if they are required to wait too long. Complaints can be fractional. See Figure 6 for the complaint distribution.

Escort Assumptions:
 - If there is a dolly available, escorts who are not currently assisting a customer will use it to collect wheelchairs left around the airport and return them to the nearest wheelchair cache.
- A given passenger has 5.5% chance of needing one of our wheelchairs. Of these passengers 25% also require an escort.
- The airline has no foreknowledge of disabled passengers' arrival. Escorts and wheelchairs are requested and assigned only when a disabled passenger deplanes or checks in.
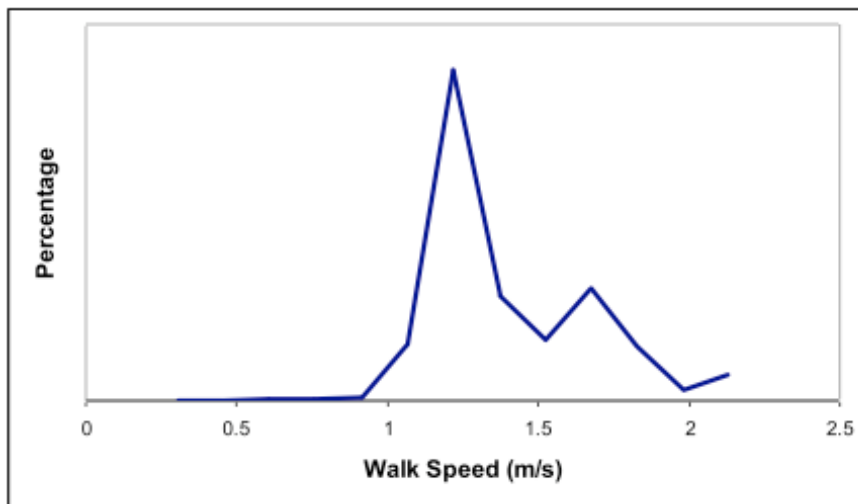
Assumptions Figures:
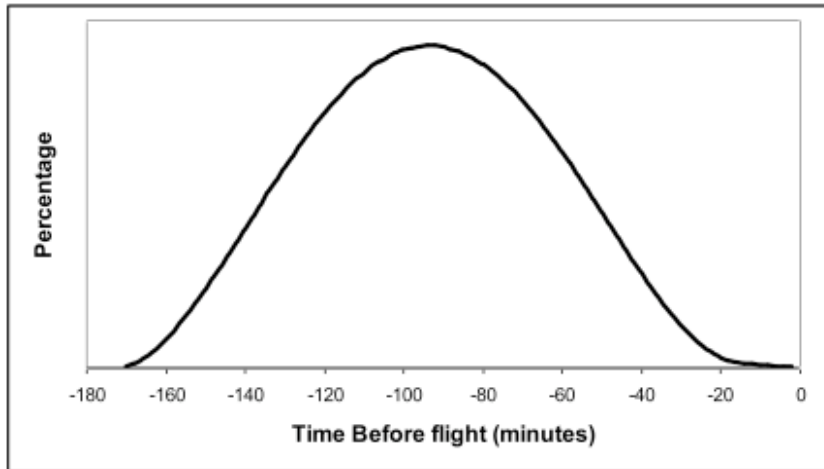

Figure 1. Walking speed distribution

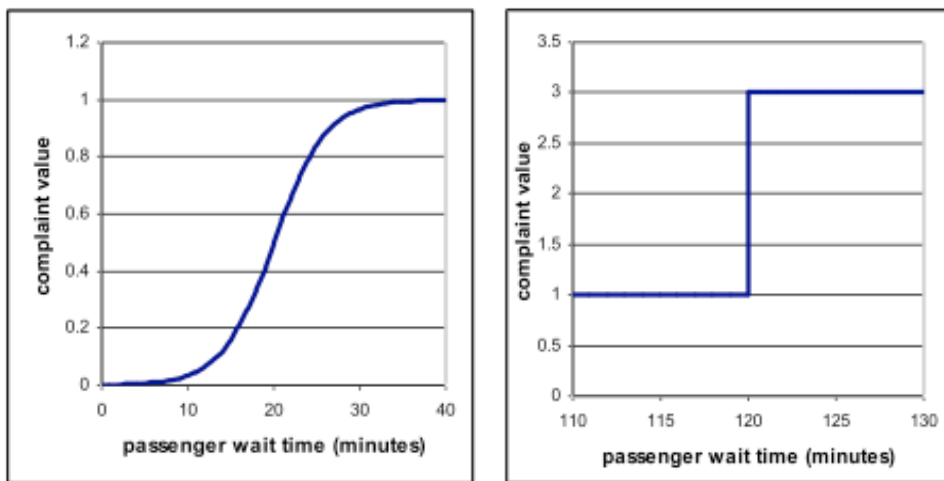Figure 2. Passenger pre-flight arrival time distribution

Figure 3. Passenger complaint distribution

Analysis of Assumption Weaknesses:

Passengers always find the shortest path to their destination. In reality, passengers may get lost or take a longer path. However, the ample signs in airports typically guide passengers along the shortest path, so this assumption is reasonable.

Our model ignores trips made to the bathroom and stores once the passenger has reached their gate. A disabled person desiring to visit the bathroom or go shopping is assumed to ask one of the gate flight attendants for assistance. In reality, the flight attendants may be busy and an escort may need to be called. Congestion due to shoppers may also be a significant factor in real airports.

Airlines often have foreknowledge of disabled passengers' needs. While airlines are not allowed to require advance notice for wheelchairs, many disabled passengers choose to alert the airline to their needs hours before their flight. Our model assumes that no passenger gives advance notice. This is useful for obtaining an upper-bound on cost and demand, but may not be realistic.

The amount walking speeds are slowed due to congestion is linear with respect to crowding. A more detailed simulation might model each passenger as a particle and perform collision detection, but this would be extremely computationally expensive.

Wheelchair Distribution Method Evaluation:

The wheelchair distribution simulation allows properties important to the cost model to emerge from the interactions of individuals with the specific airport environs, leading to a more detailed description of the effects of changing parameters of the wheelchair distribution approach. In order to evaluate the overall fitness of a given method of wheelchair distribution, the total cost incurred (per unit time) by the use of the method was determined, along with several other metrics. The total operating cost for a distribution method has several components:

e_T = Gross hourly cost for all escorts in model

w_T = Gross hourly cost for all wheelchairs in model

d_T = Gross hourly cost for wheelchair dollies in model

f_T = Gross hourly cost incurred from flight delays

c_T = Gross hourly cost incurred from disabled customer complaints

Then the total operating costs $C = e_T + w_T + d_T + f_T + c_T$

These components are derived from the simulation as follows:

e_T - Escort base wage times number of escorts. For our model, base wage is set at $13 per hour.

w_T - Total wheelchair cost is composed of cost from manual wheelchairs and cost from electric wheelchairs. For our model, normal wheelchair cost is assumed to be $0.10 per hour, while electric wheelchair cost is $0.25 per hour for fixed costs, and $0.17 per hour of actual use due to battery maintenance (See Appendix 3).

d_T - For our model, each wheelchair dolly has an hourly cost of $0.02 (See Appendix 3).

f_T - The average cost for each flight delay is assumed to be $256 (See Appendix 3).

c_T - The average cost for each complaint is assumed to be \$60. The total possible complaints were computed by summing the complaint values for each disabled passenger in the simulation, where were obtained as a function total time that passenger was kept waiting for an escort:

$$c_T = \begin{cases} 1 & \text{when } t < 120 \\ \dfrac{1 - e^{-(t-20)/3}}{3.0} & \text{when } t \geq 120 \end{cases}$$

**Wheelchair Distribution Method:**

In order to provide wheelchair and escort services to the disabled, wheelchair are stored in wheelchair banks in some airport nodes. When a disabled customer requests a wheelchair at check-in or after deplaning, the nearest escort is paged to obtain a wheelchair from a nearby wheelchair bank. If the disabled passenger does not require an escort, they can then make their own way to the gate, and the escort is free to perform other duties.

Airport Simulation Test Scenarios:

We based our simulation airport on Spokane International, a three-concourse airport servicing about 9000 people a day(http://www.spokaneairports.net). A detailed terminal map can be downloaded from the airport's website, which was used along with a less detailed map indicating scale to create a graph with realistically sized nodes. The airport's 24 gates, 2 security checkpoints, 2 baggage claims, and 2 ticketing counters were assigned to nodes.
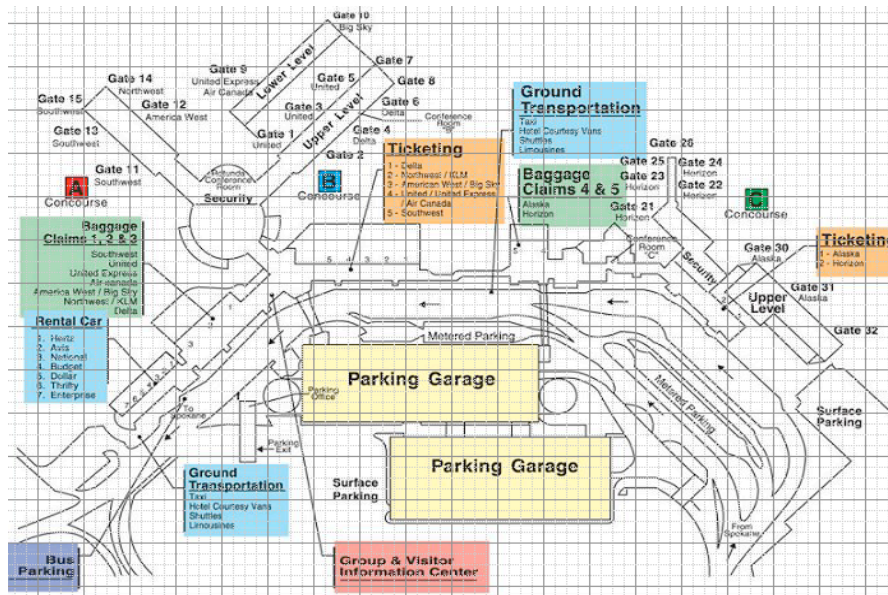
Spokane International Terminal Map
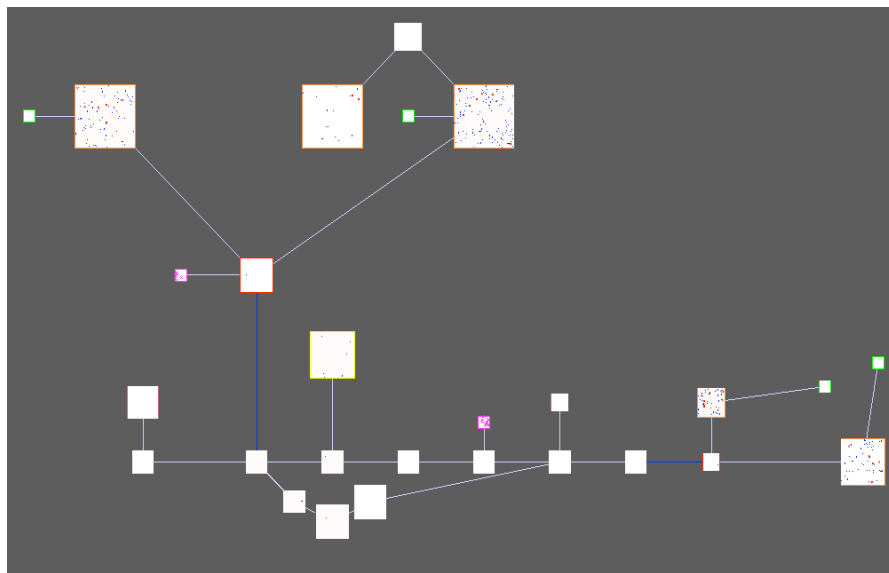
Screenshot from the simulation

The incoming flights to our test airport were randomly distributed as discussed in [x section] and the plane size was uniformly selected from [30, 120] to reflect the range of small and large planes serviced by Spokane International. The following parameters remained constant for each run of the simulation:

Number of check-in lines: 30
Number of security lines: 5
Percentage of passengers that are disabled: 5.5%
Percentage of disabled passengers requiring an escort: 25%
Percentage of disabled passengers requesting an electric wheelchair: 50%
Escort wages: $13/hr
Cost of wheelchair: 10 cents/hour
Cost of electric wheelchair: 17 cents/hour
Cost of electric wheelchair batteries: 23 cents/hour
Cost of dolly: 3.5 cents/hour
Cost of complaints: $60
Cost of missed flights: $256

Testing Wheelchair Distribution Method:

Wheelchair distribution at Spokane International was simulated first as an integrated service providing wheelchairs for disabled passengers on all flights, and second as an airline service for only Alaska/Horizon customers (Gates 21-25, 30-32). The simulations were run 150 times each with linearly distributed random parameter values and the run with the lowest total cost was used as a basis. The parameters changed and their value ranges were:

Total escorts: 1-35

Total manual wheelchairs: 25-125

Total motorized wheelchairs: 25-125

Total dollies: 1-10

For the full-airport service method, two closet placements were considered: one wheelchair closet area near check-in and the addition of another closet near concourses A and B. The total number of wheelchairs, number of escorts, and number of available wheelchair dollies were then tweaked randomly by the simulator to find near-optimal solutions, after which the parameters were tweaked by hand for further optimization.

For the one-airline service method, only one closet placement was tested, but the model parameters were tweaked by hand to discover the most cost-effective solution.

Test Results and Discussion:

The optimal parameters from the one airline, full airport, and full airport - 2 wheelchair cache methods are given below. The hourly cost and cost per disabled passenger were determined from the average of 20 runs of the optimal parameters for the given model.

Airport Model	# of wc caches	# of Escorts	# of manual wcs	# of electric wcs	# of wc dollies	Hourly Cost	Cost / Disabled Passenger
Full	1	8	48	38	3	$214.96	$6.96
Full	2	8	48	38	3	$224.59	$7.27
1-Airline	1	2	24	19	3	$60.31	$6.54

These data indicate that using a per-airline approach is optimal, given our model assumptions, and employing 2 escorts and a fleet of 24 manual wheelchairs, 19 electric wheelchairs, and 3 wheelchair dollies, leads to the most cost-effective service of disabled passengers.

Our simulation of Spokane International averages 418,030 passengers per month, which is a factor of 1.5 larger than Spokane's number of 278986 for December 2005 (obtain from the airport website, given above). Since our simulation attempts to simulate peak days for maximum effect, this is a reasonable emergent value.

Hourly cost analysis of the optimal solutions provides insight into the balance obtained by the solution. The optimal parameters generally do not lead to any costly flight delays, but the figures below show that in both the one airline and full airport optimal methods, the total cost is relatively fixed until peak hours cause delays in escort response time leading to customer dissatisfaction. This indicates that the number of resources are indeed minimal for normal operation, but if operation is expected to vary widely at a given location, it may be useful to have extra wheelchairs and support staff who can act as escorts in an emergency, since cost balloon rapidly.

Full Airport Method - Temporal Relationship Between Number of Disabled Passengers,
Average % of Time Escorts are working, and Total Cost
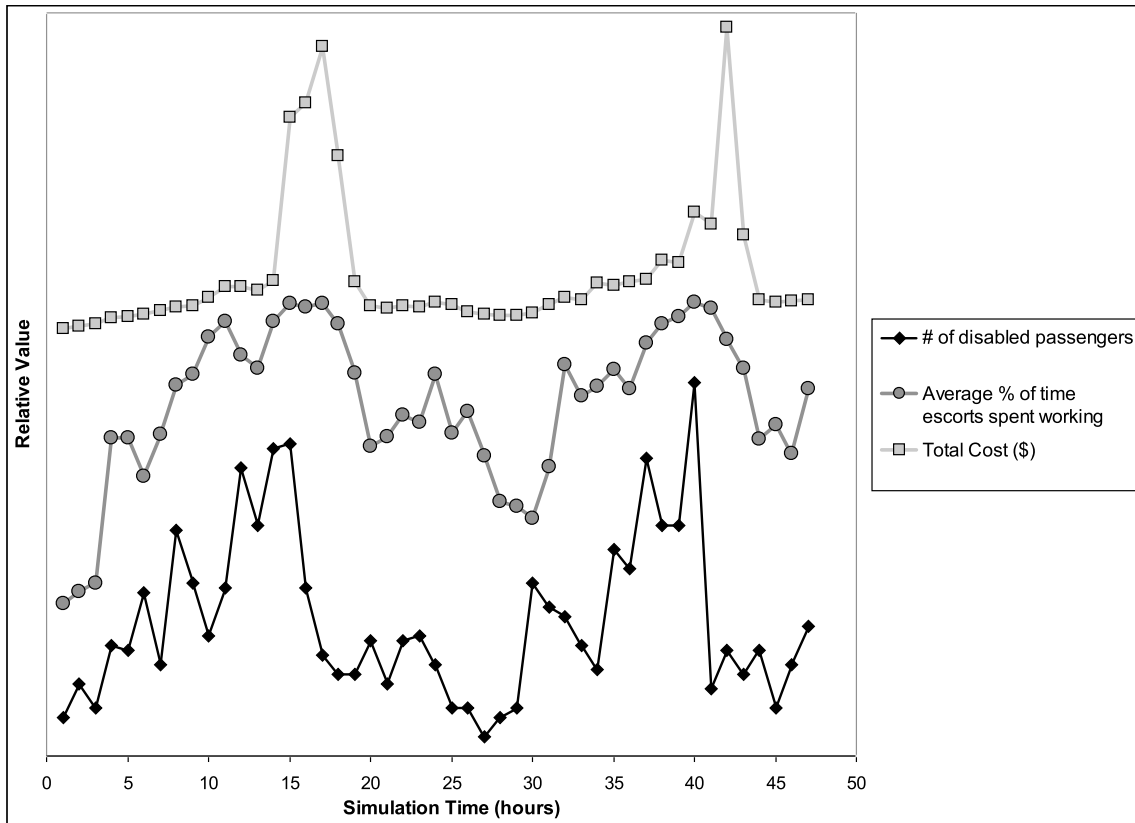
One Airline Method - Temporal Relationship Between Number of Disabled Passengers, Average % of Time Escorts are working, and Total Cost
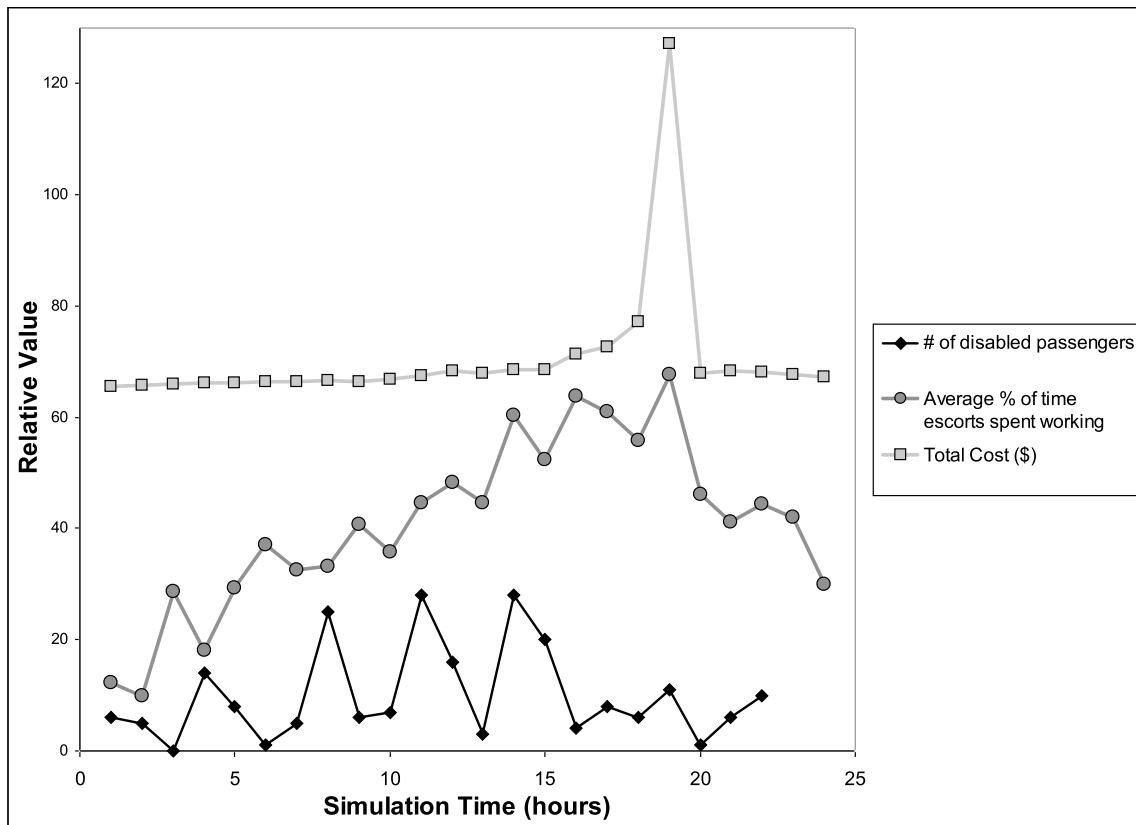
Stability Analysis:

To test simulation stability around the optimal points, the optimal parameters for each method where run 20 times for four days each run. The results follow:

Distribution Method	Mean Total Cost	Std. Deviation
1-airline, one wc	$1447.53	$330.58
full airport, one wc	$5159.13	$3157.98
full airport, two wc	$5390.33	$1346.52

These standard deviations are very large, and are likely accounted for by the high cost of unlikely random occurrence of bad flight delay or very unhappy customers. Because the system is unstable near optimal points, more computing power is necessary to find optimal points with certainty, and we recommend have a contingency plan, such as extra

wheelchairs in a shed or employees who could temporarily double as an escort during high-traffic periods.

Cache Model

Overview:
Examining one stationary cache of wheelchairs and a surrounding area assigned to it, we came up with a mathematical model of wheelchair flow in and out of a small region with space for one wheelchair closet. The airport is be serviced by a network of caches supplying their surrounding regions exclusively. Adjacent regions are able to share wheelchairs between themselves. Probabilities of passengers needing wheelchairs will be used to evaluate wheelchair usage in the average statistical case.

Assumptions:
1. Carting wheelchairs around inside their area takes a negligible amount of time, but transferring wheelchairs between areas does take time. This assumption is based on airport designs that cluster gates together at the ends of concourses.
2. If one area has a depleted supply of chairs, the other areas will have surpluses. Further, wheelchairs delivered to the area are added to the cache, regardless of where they came from--this implies sharing of wheelchairs between different airlines.
3. If we stock wheelchairs for a peak period of traffic, we can leave them where they are until the next day and there will be an ample supply for the lower-traffic periods.
4. A high degree of symmetry is assumed in the airport, namely that just as many people in airline wheelchairs are coming to our area as to the others, and just as many of our people need to transfer as do those in other areas.
5. Independence of the travelers is also assumed--each traveler, disabled or not, makes his decision which flight to take without regard to any other passenger.

Weaknesses of assumptions:
1. Depending on the airport layout, the most favorable spacing of caches may have a several-minute internal passage time, and gates on the edges of adjacent areas may be in reality quite close. Evenly spaced gates along straight concourses don't cluster very naturally.
2. Excepting wheelchair theft or decay on a grand scale, our total number of wheelchairs is constant. Given special circumstances that retard the redistribution of the chairs, however, it is possible to have an unplanned, general shortage. Given the frequent inter-airline traffic at multi-carrier airports, the most economically sound way to deal with wheelchairs is to treat them as interchangeable--at the end of the day, Epsilon Airlines would be entitled to the stock number of wheelchairs it has independently provided, but the hassle of sorting out chairs based on original owner is unnecessary. The main snag to be worked out in this scenario is determining responsibility for maintenance costs.

3. There may need to be some after-hours shuffling and cache restocking. "Busy time" is modeled with a finite, uniform duration, and if there's even one chair left in the cache at the end of that time period, it's called good, though in reality the supply may be depleted to the point that it is insufficient for even the reduced traffic to come.

4. A major weakness in the model is the lack of weights for frequency of wheelchair loss and return. In a more refined analysis of a specific airport, the assumption that the probability of wheelchairs leaving is identical to that of them arriving would be replaced with an empirical weight based on flight transfer distributions, and each possible cache site in the airport would be modeled with these data in mind.

5. Persons with disabilities who have researched their trip beforehand can be expected to cluster to airlines and airports with better histories of providing customer service to other people with limited mobility. In addition, special events such as conventions can cause a higher concentration of people needing wheelchairs then would otherwise be expected.

Restrictions:

The model is based on discrete cycles--they were assigned to be hours, but are easily adjustable for scale. The mathematical interpretation of a cycle interprets everything happening inside of it as simultaneous, so the real-world occurrence that someone must wait for up to a cycle before they get their wheelchair is ignored, since in the model there is no time delay between the arrivals of the person and the chair. In the subsequent cost analysis, no data is given regarding escorts for wheelchairs-- they are assumed plentiful. The restocking scenario assumes chairs are available from neighboring caches, but no wheelchairs from the cache in question are ever poached to stock other areas. The outcome is a general rubric and does not apply to any specific airport layout, but rather can be used in analyzing specific floor plans.

Variables:

Independent:

N: People deplaning in the area per hour - a measure of traffic flow and airport size

n: Number of wheelchairs in the cache

p: The probability that any given passenger will require an airline wheelchair

Dependent:

p_x: The probability that x passengers in the area will require a wheelchair at the same time

d: The probability that more wheelchairs will be needed than are present in an area, depleting the cache

E: The expected time at which a cache will run out of chairs, in cycles

R: The frequency with which a cache must be restocked to keep from running out of chairs assuming the average case, in cycles

For a derivation and discussion of the values used for N and p, see Appendix E1.

Equations:

(1) $p_x = \binom{N}{x} \pm p^x (1-p)^{1-x}$

(2) $d = \sum_{k=0}^{\infty} \left(p_{n+k+1} * \sum_{k=0}^{\infty} p_k \right)$

(3) $E = d^{-1}$

(4) $R = \left\lceil \dfrac{h}{E} \right\rceil$

A computer program was written to evaluate all nontrivial calculations and sums; it is given in Appendix E2.

Calculations:

Let all potential air travelers form a pool, and pick one at random. The probability that he will need an airline wheelchair while traveling is d. The pool is large enough that the first choice does not in any significant way affect its remaining members, so for each subsequent pick up to N, the probability of picking someone requiring a wheelchair remains constant at d. The probability that the group we picked has x people is, then,

binomially distributed, that is, is equal to $\binom{N}{x} \pm p^x (1-p)^{1-x}$. Assigning the picked group to

the group likely to come into our area, we come up with (1).

In order for a cache to run out of wheelchairs, there needs to be more people deplaning in the area who need wheelchairs then people entering the area in airline wheelchairs. For an area deplaning N passengers, the probability that (n+k) wheelchairs are needed is the probability that (n+k+1) passengers on board will be using wheelchairs, since one person gets to store his own wheelchair on board, or

$$\binom{N}{n+k+1} (d)^{n+k+1}(1-d)^{N-(n+k+1)} .$$

The probability that k or fewer wheelchairs enter the area could be calculated for an airport with N people and F equally-trafficked areas outside of our terminal as

$$\binom{N}{Fk} (d)^{Fk}(1-d)^{N-Fk}$$

if we were to take the assumption that the wheelchair users will be equally distributed amongst the terminals. This would be, in effect, the probability that (Fk) people need wheelchairs, and we assume that exactly k of these end up in our area, which represents 1/F of the airport. An easier calculation, however, and one that will better account for random distribution of individuals makes the alternate but similar assumption that our area is a microcosm of the airport as a whole. This is based off our prior assumption of gross symmetry and independence of disabled travelers from one another. If we have 1/F times the traffic of the airport, then we will have 1/F times the total people and 1/F times the people requiring wheelchairs in our gates. In one cycle, then, we get about 400 people, so we assume the other terminals service about 400F folks. If l percent of passengers leave the areas they enter, then 400l will leave us. (400Fl) will leave from the other terminals and 1/Fth of them come to us—that's 400l persons, the number we exported.

Based on this, a simpler calculation will simply be to assume that the probability of getting k wheelchairs is equal to the probability that k wheelchairs leave us. (This is where weights would be accounted for in a more refined model based on a specific traffic pattern.)

d is the probability that, for any k, ($n+k$) wheelchairs leave and k or fewer wheelchairs will return. Since people make the decision to leave or enter our area independently of one another, the probability that ($n+k$) wheelchairs leave in the same cycle that k or fewer wheelchairs arrive is expressed by the product of the two probabilities, thus:

$$d = \sum_{k=0}^{\infty} \left(P[(n+k) \text{ wheelchairs leave}] * P[k \text{ or fewer wheelchairs arrive}] \right)$$

$$= \sum_{k=0}^{\infty} \left(P[n+k+1 \text{ people need wheelchairs}] * P[k \text{ or fewer people need}\right.$$

wheelchairs$])$

(2) $= \sum_{k=0}^{\infty} \left(p_{n+k+1} * \sum_{k=0}^{\infty} p_k \right)$

This could be expressed as an integral. The individual summands should exponentially decrease, as is not only intuitive but also necessary for a finite result. We could model them using the normal distribution, but since that would give us an integral that we would have to approximate anyways, we'll stick to a discrete view of things.

A quirk of this calculation is that it the chance of depleting a store of 0 wheelchairs if we have 100 passengers deplaning at our area and 5.5% of our passengers require wheelchairs is 56%. If a wheelchair comes in during the time cycle it is needed, it doesn't count as if the gate were short on chairs. In airports where the time any one passenger occupies a chair is high and the people are trusting of statistics, this result actually does make some sense. True, the passenger in our area may have to wait for his chair to come rolling in, but the time involved should be minimal. At any rate, it should be faster than sending an attendant to another terminal to pick up and return with a free chair. Thus, if there were some way to know that the chair were actually on its way to our area, such as a simple walkie-talkie system for the escorts and gate attendant ("We need two chairs at Gate A3, anyone coming our way or do we have to send someone to pick them up?"), the more efficient method to use would be to wait for the chair to come on its own.

Since the likelihood of gaining chairs in this unweighted model is the same as the likelihood of losing chairs, rather than trace all possible combinations of chairs at the end of every cycle, it is acceptable to assume the cycles are independent of one another. It is true that the probability of ending up with exactly n chairs at the end of each cycle is quite small; however, the probability of ending up with more and thus buffering ourselves again future shortages is the same as the probability of ending up short and having a handicap in the future. Thus, good and bad cases cancel themselves out in the aggregate when what is primarily considered is the average occurrence. This allows us to use the geometric distribution to examine relative probabilities of first failures. The probability

that we first run out in cycle x is simply $(1-d)*d^{x-1}$, so the probability of running out no

sooner than cycle x is $\sum_{k=x}^{\infty}(1-d)*d^{k-1}$.

For the analysis of the results given by this model, it is useful to know how often a cache will have to be restocked, since employee time is expensive. The expected value for time between restocking is calculated as follows.

 E[chairs run out] = 1*P[chairs run out] + 0*P[chairs don't run out]

$$= d + 0*(1-d)$$

$$= d \text{ times per cycle}$$

thus:

(3) $E = d^{-1}$ cycles per restocking

How often caches are restocked during a given active period is determined by the frequency with which they run out. Chairs will, on average, run out d times per cycle (see above), and thus be restocked d times per cycle, provided d is an integer, which it almost certainly is not. If there are E cycles between restocking, and we're looking at a period of h cycles, the intuitive answer would be h / E restockings in this time span, but again, this is rarely an integer. This integer should be truncated, rather than rounded up--if we expect to run out every 1.9 cycles, and we're looking at a 2-cycle period, we won't have time to run out in the average case, though our supply ma well have dwindled. h / E is here 0.95, but we can round down to 0 because it is not expected that the cache will fully run out. Thus,

(4) $R = \left\lfloor \dfrac{E}{h} \right\rfloor$

Cost Analysis: Cache Model

The equations were used on four different models of airport traffic: 100, 200, 400, and 800 passengers arriving per cycle. A quick compilation of data is below.

For a small area with 100 people per cycle:

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
p	0.56	0.43	0.32	0.22	0.14	0.08	0.04	0.02	0.01	*	*	*	*	*	*	*

E	1.8	2.3	3.1	4.5	7.1	12	25	50	100						

For a medium area with 200 people per cycle:

| N	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
P	0.54	0.46	0.37	0.29	0.22	0.16	0.11	0.08	0.05	0.03	0.02	0.01	*	*	*	*
E	1.9	2.2	2.7	3.4	4.5	6.2	9.1	12	20	33	50	100				

For large area with 400 people per cycle:

| N	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
P	0.53	0.47	0.41	0.35	0.29	0.24	0.20	0.16	0.12	0.09	0.07	0.05	0.04	0.03	0.02	0.01
E	1.9	2.1	2.4	2.9	3.4	4.2	5.0	6.2	8.3	11	14	20	25	33	50	100

For a very large area with 800 people per cycle:

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
P	0.52	0.48	0.43	0.39	0.35	0.31	0.27	0.24	0.21	0.18	0.15	0.12	0.10	0.09	0.07	0.06
E	1.9	2.1	2.3	2.6	2.9	3.2	3.7	4.2	4.8	5.6	6.7	8.3	10	11	14	17

*less than 0.01
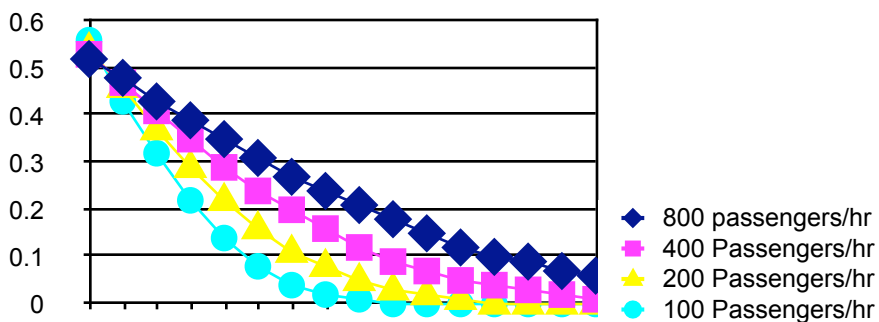
The graph indicates that, for smaller airports, the marginal probability of depletion is greater than that for higher-traffic areas. This implies that the value of a single chair is higher for a small airport than it is for a large one, which is helpful, as smaller airports generally have smaller budgets and capital is at a premium.
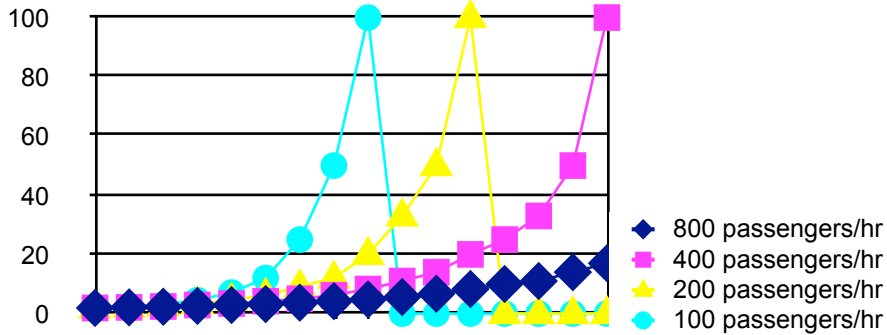
The high final slopes of the expected time till cache depletion suggest that the addition of just one or two wheelchairs over the calculated amount for maximizing efficiency would strongly buffer the caches against premature depletion. For a struggling airline that can't afford the costs and image problems associated with complaints and delayed flights, an investment into a few wheelchairs, one or two per cache, would be a wise move against future costs when an area becomes unforeseeably busy or chairs break down.

A cycle was set to one hour and costs calculated in Appendix E3 were ascribed to a time span of 8 hours, a reasonable estimate for the length of a busy time of day for a business. It is assumed that every time an employee left to restock the cache, they brought back the number of chairs originally stored there. The time associated with rounding up spare chairs is more or less an arbitrary guess, bounded on the outside by 45 minutes, since rounding up many chairs is more efficient than rounding up a few, and on the low side by 10 minutes, since an assumption for the model is that the areas have some basis in architectural reality and things outside our area are a fair distance away. This will, of course, depend on the distance between gates, the relative availability of chairs in a particular region of the airport, the number of chairs gathered, and foot traffic between areas.

The costs calculated are not the total costs associated with the wheelchairs--most notably, the costs of escorts are left out. The costs below are for purposes of internal comparison only. It is assumed that escort-hours are a function of the needs of the passengers and not the distribution of wheelchairs throughout the different caches; as such, it is not a factor in the comparison of cache models.

For 100 passengers/hr:

Chairs	0	1	2	3	4	5	k>5
Restocking frequency per 8 hrs	4	3	2	1	1	0	0
Cost of restocking per 8 hour day (min per trip)	$8.70 (10)	$6.50 (10)	$6.50 (15)	$4.30 (20)	$6.50 (30)	$0	$0

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Cost of having chairs per day of 5-yr life (assuming manual)	$0	$1.10	$2.20	$3.30	$4.40	$5.50	$1.10*k
Cost per day	$8.70	$7.60	$7.70	$7.60	$10.90	$5.50	$1.10*k

For 200 passengers/hr:

Chairs	0	1	2	3	4	5	6	k>6
Restocking frequency per 8 hrs	4	3	2	2	1	1	0	0
Cost of restocking per 8 hour day (minutes per trip)	$8.70 (10)	$6.50 (10)	$6.50 (15)	$8.70 (20)	$6.50 (30)	$7.60 (35)	$0	$0
Cost of having chairs per day of 5-yr life (assuming manual)	$0	$1.10	$2.20	$3.30	$4.40	$5.50	$6.60	$1.10*k
Cost per day	$8.70	$7.60	$7.70	$12.00	$10.90	$13.10	$6.60	$1.10*k

For 400 passengers/hr:

Chairs	0	1	2	3	4	5	6	7	8	k>8
Restocking frequency per 8 hrs	4	3	3	2	2	1	1	1	0	0
Cost of restocking per 8 hour day (minutes per trip)	$8.70 (10)	$6.50 (10)	$9.80 (15)	$6.50 (20)	$13.00 (30)	$7.60 (35)	$8.70 (40)	$9.80 (45)	$0	$0
Cost of having chairs per day of 5-yr life (assuming manual)	$0	$1.10	$2.20	$3.30	$4.40	$5.50	$6.60	$7.70	$8.80	$1.10*k
Cost per day	$8.70	$7.60	$12.00	$9.80	$17.40	$13.10	$15.30	$17.50	$8.80	$1.10*k

For 800 passengers/hr:

Chairs	0	1	2	3	4	5	6	7	8	9	10	11
Restocking frequency per 8 hrs	4	3	3	3	2	2	2	1	1	1	1	0
Cost of restocking per 8 hour day (minutes per trip)	$8.70 (10)	$6.50 (10)	$9.80 (15)	$13.00 (20)	$13.00 (30)	$15.20 (35)	$17.30 (40)	$9.80 (45)	$9.80 (45)	$9.80 (45)	$9.80 (45)	$0
Cost of having chairs per day of 5-yr life (assuming manual)	$0	$1.10	$2.20	$3.30	$4.40	$5.50	$6.60	$7.70	$8.80	$9.90	$11.00	$12.10

Cost per day	$8.70	$7.60	$12.00	$16.30	$17.40	$18.70	$23.90	$17.50	$18.60	$19.70	$20.80	$12.10

For 100 and 200 - passenger areas, the cheapest distribution of wheelchairs is 5 and 6, respectively, per area. This is the point at which, in the average case, the cache will last through the day. For a 400-passenger area, having this amount (8 chairs) is only nominally more expensive than the next-lowest option, which is 1 chair and frequent restocking. The difference is a little bigger for our behemoth area of 800 passengers an hour, with a 60% higher cost than providing but one wheelchair per area.

What is lacking in this analysis are the peripheral costs associated with not having enough wheelchairs. Aside from the miserable traveler who must wait at his gate for a chair to free up, monetarily we have already calculated a conservative estimate for the price of one delayed flight to be around $250. If one delayed flight costs $250, and we're saving $4.50 a day, fewer wheelchairs only become more economical if they cause a delayed flight fewer than about once every two months. Since the models are calculated in one-hour chunks, delays of individual passengers lasting less than this can be hidden away in the calculation as if there were no delay at all. Thus, the odds of delaying a flight with such a thin margin of error chair-wise (one chair fewer and we're out) are intuitively non-negligible, and it is best to err on the side of caution in determining the optimal chair count.

Conclusion:
We can now deduce that is it most cost-effective, as well as quite customer-friendly, to stock wheelchairs in such number that, in the average case, they will not require restocking in the course of one busy day. (In an *h*-cycle busy period, use that *n* which

results in *hd*<1; that is, solve $\sum_{k=0}^{\infty} (p_{n+k+1} * \sum_{k=0}^{\infty} p_k) < h^{-1}$ numerically for n.)

As a note, this will hopefully provide more wheelchairs than will be needed at any one time. That leaves plenty of freedom for maintenance and repair and, when extreme cases do occur that necessitate an extraordinary number of chairs to be in use at once, such as a snowed-in airport with a whole day's passengers stuck in the airport for a long period of time, there will be enough chairs to go around.

One important factor heretofore glossed over is the amount of space that can be practically allotted. If there are a bunch of gates in one area, then technically it would be cheapest to have them all serviced by the same wheelchair pool, but if there simply it's enough room for a closet of requisite size in the area, or it is deemed to be a fire hazard,

the area should be split up into the largest segments possible to share chair caches that can be stored in the limited space.

Data Quirks:
An interesting thing to note is that, for small numbers of wheelchairs, the prices are the same between an area that services 100, 200, 400, or 800 people an hour. This confirms the intuitive notion that one wheelchair cache can be most efficiently used by the largest area that can access it in a reasonable amount of time if we aren't expecting a large demand. Putting a small cache at each gate is intuitively less efficient than a big cache for a cluster of gates, but for configurations such as those above, perhaps for a small airline with very little capital to buy chairs, it is shown further that a small cache at a centralized place for a small number of gates is less efficient than the same small cache being used by a larger number of gates, assuming each has reasonably quick access. A deciding factor in wheelchair distribution, then, is how gates in an airport are clustered together.

In the later discussion ("Projections") on the effects of different values for p, it can be seen that the mapping of p onto the optimal value for n approximates a simple linear contraction. Since changes in p affect the results up to a constant of proportionality, we can be assured that the accuracy of p is not the cause of an undue amount of error.

Projections for Future Costs

Sensitivity of the Cache Model to Changes in the Average Demand of Wheelchairs: Assuming that the cheapest way to stock wheelchairs continues in the future to be the number at which it is expected an entire shift will pass without the supply running out, in order to predict future costs we shall now examine the effects of rising and falling rates of travelers who need wheelchairs. (For the derivation of the percentages, see Appendix E1: Projected Values.)

If 5.7% of passengers require wheelchairs (projected for 2010):
The results are the same as for a 5.5% wheelchair use rate, thus we can expect a negligible increase in wheelchair costs over the coming 4 years

100 people/hr:	200 people/hr	400 people/hr	800 people/hr
5 chairs	6 chairs	8 chairs	11 chairs
$5.50 daily	$6.60 daily	$8.80 daily	$12.10 daily

If 7.2% of passengers require wheelchairs (projected for 2020):

100 people/hr:	200 people/hr	400 people/hr	800 people/hr

5 chairs	7 chairs	9 chairs	13 chairs
$5.50 daily	$ 7.70 daily	$9.90 daily	$14.30 daily

If 8.7% of passengers require wheelchairs (projected for 2030):

100 people/hr:	200 people/hr	400 people/hr	800 people/hr
6 chairs	7 chairs	10 chairs	14 chairs
$6.60 daily	$ 7.70 daily	$11.00 daily	$15.40 daily

Wheelchair usage is expected to increase at least 58% to a total of 8.7% or more by 2030. This elevated need for wheelchairs affects all of the airport in a somewhat buffered manner, with the increase in wheelchairs needed only rising by about 20-25% for the four airports examined. Part of this is the discrete nature of the model. Part of it is also that, with a larger number of wheelchairs and wheelchair users, the chance of a freak run on wheelchairs resulting in the depletion of our cache is more remote.

The costs associated with operating the wheelchairs that are dependent on how much time they spend in use--primarily escorts' salaries and maintenance--can be expected to increase in direct proportion to wheelchair need, so total costs of wheelchair operation can also be reasonably expected to increase 20-25% by 2030.

Recommendations
The data found for optimization of the specific airport modeled (2 escorts, 3 wheelchair dollies, etc.) does not directly extrapolate to other airport models, but does demonstrate the ability of the method to find specific optimal parameters. As optimal solutions become unstable during high-traffic periods, the most cost-efficient plan should come with backups--people to double as escorts, back stock of wheelchairs, etc.--to ensure greatest efficiency. Though the simulation was only run for one airport model, and thus only simulated one size of airport, it agreed with the findings of the cache model with regards to optimal setup of wheelchair caches.

The computer simulation came up with an optimal model that had wheelchairs shared only between gates rented by a single airline and serviced by a single cache. In the model, these airlines rented space in clumps. This corresponds to the case in the cache model, created independently of the simulation, in which the most efficient configuration was found to be a single cache in a large area, with as little long-distance swapping of wheelchairs as possible. From this agreement between models, we can conclude that the most efficient way to distribute wheelchairs is in single, well-stocked caches serving sections of airport.

Appendix T1:
Dijkstra's Algorithm

To find the lowest cost path from vertex A to vertex B, we use Dijkstra's algorithm. Dijkstra's algorithm finds shortest paths from vertex A to all other vertices in nondecreasing order of length. The first path is from vertex A to vertex A with length 0. Then, among all of the vertices that can extend a shortest path already found by one edge, it selects the one that results in the shortest path. It continues to apply this greedy rule until vertex B is found, at which point the shortest path is extracted by examining the tree of decisions made by Dijkstra's algorithm. For our implementation of Dijkstra's algorithm, see the class method Person::computeVisitList.

Appendix E1:
Discussion of Assumed Variable Values

N:
The values used to represent the total number of people in an area at any given time are drawn from airport timetables and airplane specs. Planes with a capacity of under 100 people are not required to be equipped to handle people in wheelchairs (US Department of Transportation, 2003) so we focus our attention on flights this big and bigger. Boeing 747s can fly with between 374 and 496 passengers (Airlines of the Web, 1999-2006) so we examined heavy traffic in increments of 400 in the interest of working with round numbers, using 400 instead of 500 since 500 seemed like the extreme upper bound on commercial passenger planes.

As of this writing, at the Portland airport, one flight comes into gate D7 at 2:17 pm, leaving at 3:30pm. Another plane comes in at 4:32 pm, then a separate flight arrives at 4:50 (Port of Portland, 2006). 28 minutes to deplane passengers and get the plane out of the runway would not be realistic for larger airplanes, but one hour would be. This is confirmed by the Dallas/Fort Worth schedule, which as of this writing had scheduled a flight arriving at gate A39 at 1:15pm and another at 2:15 pm. Thus, for larger flights, we assume at least an hour between arrivals at a certain gate. Though the calculation was figured in cycles, an abstract measurement of time, for the analysis it was necessary to assign some actual reference value. Hours were used as it seemed like the smallest uniform time step in which there could reasonably be assumed to be an incoming flight to an area. Multiple-gate areas would decrease the time step, but hours are good numbers with which to work.

p:
Present value:

About 53 million (National Organization on Disability, 2004) of the US's roughly 298 million (US Census, 2005) are classified as living with disabilities. That makes for about 17% of the population. Of these, 17% report that they require but do not personally have assistive technology, and of these, 19% report lacking a motorized wheelchair or scooter and 6% report lacking a manual wheelchair. (National Organization on Disability, 2004) This is the segment of the population for which we are required to provide mobility assistance. The above figures imply that 0.55% of the population needs motorized wheelchairs they don't have, and 0.17% need to have manual wheelchairs provided.

Only 60% of people with disabilities take long distance (100 miles and more) trips, compared with 76% of people without disabilities (Bureau of Transportation Statistics). Assuming there are no significant demographic differences between people with disabilities at large and the subset of people who require wheelchairs, we can expect 0.075% of our passengers to request a manual wheelchair and 0.24% to request an electric (or a manual wheelchair with an escort to push, depending on availability).

Perhaps more significantly, people who may not consider themselves to need a wheelchair but who have significant difficulty walking a quarter mile, which make up 6 or 7% of the US population (Bowe, 2005), (National Center for Health Statistics, 2005), will require significant assistance if they must travel quickly between far-removed terminals. We can assume that we will have to provide assistance to all of these people, in the form of wheelchairs and/or escorts, but only for connections that take the passenger between terminals. If we treat these people as having disabilities, then statistically they are only 0.79% as likely as people without disabilities to travel, leaving 4.7% - 5.5% of our air traveling population with such a limitation. People in such a situation are also likely to be unaccustomed to piloting a wheelchair, as they likely do not have their own, and are may uncomfortable with an electric, in which case they might request an escort.

This figure -- somewhere around one in twenty -- comes off as larger than intuition would dictate. Statistics useful for refining this amount are frustratingly hard to come by in the public domain, but the only source we could uncover implied 5.8% of people traveling by air will need to borrow wheelchairs (Haseltine, 1996-2004), which confirms our number. A better estimate would be available should we get the contract and gain deeper access to Epsilon Airlines' records.

Projected Values:
With the aging population and improvements in health care allowing disabled people to lead longer, fuller lives, the demand for airline tickets for people with limited mobility is expected to rise for the majority of the foreseeable future. While technological developments make it hard to predict exactly how our aging population will sort out demographically with respect to disabilities, we can do our best to make an educated guess as to how our need for wheelchairs will change.

Interestingly, the percentage of people aged 65 and over who are disabled, both institutionalized and non-institutionalized, is expected to drop. For persons who are not living in nursing homes and the like, the current prevalence of limitations to daily activities of 22%is projected to dip slightly to 21% by 2010, hitting 20% by 2020 and holding there through 2030 (Goldman). Assuming that the percentage of people whose disability includes limited mobility follow a similar trend, as do other age groups, the upper estimate of the percent of people needing wheelchairs from the airport could be expected to drop from 4.7% to 4.5% by 2010, then to 4.1% for 2020-2030.

The question becomes, then, not only one of US demographics, but of those people actually flying. Seniors at the moment account for 80% of all luxury travel (Suddenly Senior, 2002). Business travel is almost entirely demanded by people under 65 (Georggi) who are still working, a demographic far less likely to be disabled than their older, usually retired, counterparts. Focusing then on the change in the demographic of people flying for pleasure, or somewhere around 57% presently for commercial airlines nationwide (Georggi), we could parse off 46% of the present market for senior travel. (This is a pretty rough estimation, since part of it is based on travel in general, not just by air.) As seniors and their disposable incomes increase, this is the segment wherein we can expect to see proportional growth.

A more refined analysis of the market (Georggi) shows people over 65 accounting for 34% of all air travelers on social trips, 31% on recreational trips, and 25% of all personal business trips. If this data can be extrapolated to the rest of the market, seniors control 32% of pleasure travel and 25% of business, or a total of 29% of the market. This is a much more conservative estimate than the above 46%, but the numbers used come from a more detailed and scientific source. We'll take 29% of the market to be the lower bound of senior control.

Disability among younger people is generally due to disease, accident, or genetics. Though treatments are constantly improving, we can expect the rate of disability due to such factors to remain more or less constant among the younger population. The more important statistic would be the increase in the size of the senior population. In 2000, people over 65 accounted for 12.7% of the population. This is expected to rise to 13.2% in 2010, 16.5% in 2020, and 20.0% in 2030 (US Census Bureau, 2004). Any change in non-senior disability rates is likely to have an overall effect of less than the rounding error in this larger figure, so we'll ignore non-senior wheelchair needs. Likewise, the decrease in senior disability rates predicted (see above) will also work out to a very small fraction, so we'll simplify our calculation by assuming that the rate of airport wheelchair use is proportional to the size of the airline market controlled by seniors, and that the size of the market controlled by seniors is proportional to the statistical representation of seniors in the US population as a whole.

	2000	2010	2020	2030
Senior share of market	≥ 0.29	≥ 0.30	≥ 0.38	≥ 0.46
% passengers needing wheelchairs	5.5%	5.7%	7.2%	8.7%

Thus, a lower bound on the percent of air travelers needing wheelchairs in 2030 is 8.7%. Assuming increased government regulations to accommodate travelers with disabilities, which would leader to a more comfortable traveling experience and a higher proportion of the limitedly mobile in the air, this number may even be higher.

Appendix E2:
Code to evaluate expressions in the cache model

```
//This program evaluates probabilities relating to wheelchair needs
//during an average day at an airport


#include<iostream>
#include<cmath>

using std::pow;
using std::cin;
using std::cout;
using std::endl;


//returns the probability that exactly x of N passengers will require a
//wheelchair if wheelchair users are randomly distributed among flyers
//default assumption: 5.5% of passengers, on average, require a
wheelchair

long double prob(int N, int x, float percent = 0.055)
{
    //calculates (N over x) * p^x * [(1-p) ^ (N-x)]

    long double Wahrscheinlichkeit = 1;

    /*starts with the bin calculation: alternates between
multiplying
    numbers in the numerator and dividing ones in the denominator
    and multiplying by the p's in order to keep the number
    manageably small*/
    for(int ii = 0; ii<x; ii++)
            Wahrscheinlichkeit *= ((percent)*(N-ii)/(x-ii));

    //multiplies in the (1-p)'s
    Wahrscheinlichkeit *= pow(1-percent,(N-x));
```

```
        return Wahrscheinlichkeit;

    }



//returns the probability of n or more people needing a chair
long double gthan(const unsigned int N, const unsigned int x, const
float p = 0.055)
{
      long double value = 0;
      for(int ii=x; ii<=N; ii++)
              value += prob(N,ii,p);
      return value;
}


//returns the probability of n or fewer people needing a chair
long double lthan(const unsigned int N, const unsigned int x, const
float p = 0.055)
{
long double value = 0;
for(int ii = 0; ii<=x; ii++)
        value += prob(N,ii,p);
return value;
}


/*returns the probability of depleting a supply of n wheelchairs in one
area
where there are N passengers arriving per cycle
note: runs in n-squared time*/
float deplete(const unsigned int N, const unsigned int n, const float p
= 0.055)
{
      float value = 0;
      for(int k = 0; k<100; k++)
                  value += (prob(N,n+k,p)*lthan(N,k,p));
      return value;
}


//calculates the probability that the a cache of n chairs will run out
on the
//s-th cycle and not before
float geom(const unsigned int N, const unsigned int n, const unsigned
int s, const float p = 0.055)
{
      float value, dep;
      dep = deplete(N,n,p);
      value = pow(static_cast<double>(1-dep),static_cast<double>(s-1))
*dep;
      return value;

}


int main()
{
```

```
//one of the calculations which was run
for(int n = 0; n<16; n++)
        cout<<"when n = "<<n<<", d = "<<deplete(800,n,0.087)<<endl;

    system("pause");
    return 0;
}
```

Appendix 3:
Evaluation of Costs

Assuming a fleet of f chairs of given type which last for 5 years apiece and are used 8 hours a day:

Initial Costs per Chair:
Manual:
Chair - $500
(Over five years: $0.05/hr)

Electric:
Chair - $1000

Batteries, chargers - $ $\dfrac{\left[\dfrac{f}{6}\right](475)}{f}$

(Over five years: $ $\left[\dfrac{\left[\dfrac{f}{6}\right](0.011)}{f} + 0.023\right]$ / hr;

for a fleet that is a multiple of 6, $0.02/hr)

Wheelchair Dolly:
$1500
(Over five years: $0.02 / hr)

Hourly Costs of Chair Use

Manual Pushed
Escort - $12
Repairs and Adjustments - $0.09

Total: $12.09

Manual self-operated
Repairs and Adjustments - $0.09
        Total: $0.09

Electric:
Repairs and Adjustments - $0.17
Battery Maintenance - $0.23
        Total: $0.40

Costs of Flight Delay
Serious enough to cause rescheduled transfers: $256

Data Compilation:

Lifespan:
(Alexander, 1994-2005).

Escort Wages:
$12/hr seemed like a good approximation at the time the numbers were being compiled, and Epsilon Airlines' standard wages will of course be substituted before any policy implementation. The later investigation into wages for maintenance personnel (see next paragraph) showed a similar salary.

Extra Batteries, Chargers:
Batteries are generally good for the life of the chair; however, the more the battery has been drained, the longer it takes to recharge, from 3 -5 hours under light use to 8-10 if it's nearing empty. (Hoffman, 2005) We'll assume that no chair should be allowed to run for more than 16 hours before recharging for fear of the battery failing while a passenger is using it unescorted, and after 16 hours the battery will be sufficiently depleted that charging will take 8 hours. Further, if the chair is only used for 8 hours before recharging, the charging will only take 5 hours. We will also assume the presence of existing maintenance personnel working for $13/hr (Wayne county), (carreerbuilder.com) with enough free time to service the fleet. With a large number of wheelchairs, such as those in a busy metropolitan airport, the time spent on maintenance may be cause for the hiring of extra staff, which would add costs not considered here. The electricity used in charging batteries is also assumed to be a non-issue; it may add up to a noticeable amount in a large airport, but will remain a small fraction of total bills.

Assuming 8 hours a day of wheelchair use--that is, during off hours, only a fraction of the total will be needed, and the rest can be serviced in rotation-- that leaves 16 hours a day to swap and charge batteries. In this time, one charger can charge batteries for 5 chairs that have been running for 8 hours apiece, at an initial cost for the charger of $400 and a per-time cost of about $13 for labor. This works out to $400 initial cost and $0.33 per hour of battery use per chair, or $5,145 over five years per chair. One charger with one spare battery charging while the chairs are in use can service 6 chairs, with an initial cost of $475 for battery and charger and still about $13 for labor. This works out to $475 initial costs and $.27 per hour of battery use per chair, or $4429 per chair over five years.

Assuming 16 hours of wheelchair use over 2 days before charging, we still have 16 hours a day to swap batteries when the wheelchairs are assumed to be in low demand, but in this time we can only charge two batteries per day. If we have one charger and no spare batteries, we can service 4 chairs continuously, two per day. With labor costs of $8 per day, this scheme works out to an initial cost of $400 for four chairs and $0.25 per chair-hour of battery use. Over five years this becomes a total cost of $3,593 per chair. If we add an extra battery to the equation, we can charge three batteries a day, servicing six chairs with one charger and one spare battery. This would make for about $11 in labor daily to change three batteries, or $0.23 per battery hour per chair. With an initial cost of $475, the total five-year cost adds up to $3821.

A simple comparison shows that the most cost-effective scheme requires one battery charger and one spare battery for every 6 wheelchairs, so the initial costs for a fleet with f

chairs is $\left\lceil \dfrac{f}{6} \right\rceil (475)$ dollars.

Dolly:
An average-looking dolly that would fit a goodly amount of folded wheelchairs (manual or electric) costs $1100 and come with a 10-year warranty. It is assumed that a model better-suited to our needs exists, but would likely be somewhat more expensive due to larger capacity and tougher build. The lifespan should be similar, so the calculations were made under the assumption that the dolly will last for 10 years. No maintenance was calculated because such a simple, robust piece of equipment with so few moving parts and such a low importance on safety shouldn't need much tweaking.

Repairs and Adjustments:
An annual average estimate for manual wheelchair repairs under personal use is $125 (Alexander, 1994-2005). The standard recommendation for such usage is a tune-up every six months; for wheelchairs that bang into a lot of things, certain adjustments can be needed every month (Hoffman, 2004). Our wheelchairs won't be custom-fitted to anyone, and as such won't need such delicate adjusting, but we can still expect doubled

maintenance costs of $250 per year. At eight hours a day, that works out to $0.09 per hour going to maintenance.

For electric wheelchairs, the average user spends $250 per year on repairs (Alexander, 1994-2005), so we can expect to spend $500. This works out to $0.17 per hour of use for maintenance.

Flight Delay:
Requiring someone to take a later flight because of an airline's inability to provide transportation to or between gates is not an option--airlines are required by the air carrier access act to provide assistance boarding and making connections. Though this may, in some instances, be cheaper than holding up an entire flight for a significant period of time, it will not be discussed due to impracticability.

There costs of a flight that is delayed enough to affect the scheduling of other flights is mostly tied up in the administrative complexities. If individual travelers have missed connecting flights, gate attendants and other staff will likely be inundated by irate customers demanding an alternative flight route be arranged. On a flight of 100 people, where 15 have missed their connections, and assuming it takes 20 minutes to sort out one person's itinerary and the staff are being paid $13 an hour, that's an initial man-hour cost of $65. If any of the people are unduly inconvenienced, e.g. they need to travel a low-frequency route and must wait overnight, they are generally given vouchers for food and sometimes lodging, which can easily match and exceed the initial man-hour cost. If the tickets are transferred to another airline, and there is a price discrepancy, then this is also a cost for the airline company to bear.

If arrival and departure gates must be rescheduled, this will take up several man-hours at least. One or more employees must be involved with finding free gates for scheduled takeoffs and landings and generally shuffling things around. Announcements over the loudspeaker must be made to ensure that the passengers already in the building know that their gates have been moved, and information desk staff will likely be busied with confused travelers trying to find their proper gates and unable to do other work.

A scenario involving a particularly unfortunate delay, where we have the $65 of time rescheduling passengers with connections, a $100 fee to transfer two people's flights to different airlines, one hour of work to reschedule gates, two hours of work to get the word out about the rescheduled gates, and four hours of work to help confused customers would end up costing the airline $256. An escort paid $13 an hour costs less than half that amount after an eight-hour day. This reinforces the importance of adequate scheduling. Serious delays should be avoided in the average case as effectively as possible, if not for customer service then for accounting.

Note: If delays occur frequently, or are planned for and scheduled, they can cut into future profits. In a study done on Netherlands airports on the value of different aspects of a flight to the customer (Ortuzar, 1998), the second strongest variable in determining which route to chose (after price) was access time to the airport. An airport one hour closer to home was worth the equivalent of 20%-40% of the ticket price for business travelers and 10%-20% for other travelers. Stopover time at transfer airports was weighted at half the importance of access, so we can infer that a half-hour increase in the expected transfer time (such as what could be engendered by passengers requiring wheelchair assistance waiting around for chairs and/or service) is worth 10%-20% of the ticket price for passengers traveling for business and 5%-10% for everyone else. (The study took into account intra-European travel as well as intercontinental; the European weight was 2/3 instead of 1/2, but since most of the intercontinental flights went to North America, this demographic seemed much more relevant.)

In a small airport in a rural destination, where there aren't many transfers to begin with and not a lot of other options, the losses are likely to be minimal. Likewise, a large airport servicing a low-frequency flight would also have a clientele with limited other options and wouldn't feel much of an effect from reasonable delays. In these cases, the most cost-effective option very well might be to under-staff the escorts, still getting everybody where they need to go, just not in such a frenetic rush. On the other hand, on flights leaving from an area with a high concentration of carriers offering similar services, where access times and ticket costs are comparable, an extra thirty minutes of layover time might be the deciding factor for consumers pricing tickets.

Appendix E4:
Cost Comparison of Escorted Manual, Self-Propelled Manual, and Electric Folding Wheelchairs

There are three main types of passengers who will require or request the assistance of a wheelchair. First are those who are non-ambulatory accustomed to independently piloting their own wheelchair. Next are those who are semi-ambulatory but unable to walk long distances. We shall assume that they are generally unable to propel their own wheelchairs, being less accustomed to such a mode of transportation. Those who have purely physical handicaps would likely be able to use an electric wheelchair or scooter independent of an escort. This leaves a segment of the population that would require an escort to push them or would simply be uncomfortable driving an electric chair.

Planes with a capacity of 100 persons or more are required by the Air Carrier Access Act to provide room for one folding wheelchair in the cabin. People accustomed to wheeling themselves around who choose to store their chairs on the plane stay in their own chairs until using the special skinny chair that fits in the plane aisle to board. These people will no more require an escort than the average walking person. If their chairs do not fold or

have spillable batteries, or if there are several on a single flight, or if for some other reason are not candidates to be brought along in the cabin, then some people may have to check their own chairs and borrow chairs at the baggage check from the airline. Again, these people will generally not require escorts, and the chairs will be left at their departure gates. All we have to require for these people is a chair that they can push themselves.

People without their own wheelchairs but with limited mobility who are able to independently navigate the gates can be given battery-powered chairs if they are comfortable. There are likely to be people unfamiliar with such a mode of transit who would prefer to be pushed in a normal chair by an escort. In this case we have two choices: push them in a chair normally used for people who power their own wheelchairs, or push them in an escort chair. The former would simplify the problem of supplying the chairs since we would have the same kind of chair for two types of passenger; the latter would be more comfortable for the escorts but involve a greater organizational complexity. (For those people who are unable to walk to their gates unescorted, we can either escort them as they ride in a power chair or have the escorts push them. Since the power chairs cost more to buy and maintain than the manual ones, and the escorts are occupied either way, it makes sense to go with the less expensive option.)

Under ordinary circumstances, both manual and electric chairs and scooters are assumed to have 6 years of useful service (Alexander, 1994-2005). Though personal use often involves loading chairs in and out of cars, shoving chairs into closets filled with other chairs constantly throughout the day is sure to increase the rate at which the frames dent, the wheels pop, and things come loose. The damage caused by people who have no financial responsibility for repairs of the chair, and no reliance on them, is also greater than that caused by people caring for their own personal property. For this reason, we assume the life of an airport wheelchair to be 4-5 years, depending on treatment. Employees who routinely shove the chairs into closets will, of course, shorten the expected life span.

(Note: Though higher-paid employees may be less likely to mistreat the chairs, both out of a sense of professionalism and because they are more easily replaced. It is, however, almost impossible to make this financially feasible from the standpoint of increasing chair life alone. Assume a $500 manual chair (ABLEDATA, 1994) is used with an attendant for a total of five hours a day. A pay raise of $0.50 / hr would, in 1000 hours, add up to the cost of a new chair. 1000 hours, however, are logged by our theoretical chair in 20 days, or under three weeks, since chairs don't take Sabbaths. In fact, assuming only an hour a day of use for a chair with a 4-year lifespan (1461 hours total), which is so low that it would almost certainly last longer if not stored in an area particularly conducive to corrosion, we can generally estimate the monetary value of adding a year to its lifespan to be $125, or about a fifth the cost of the chair. In order to not exceed this amount in money spent on employee raises, we calculate that the cost per hour of better

chair treatment is (125 dollars / 1461 hours) = $0.09 per hour. On the other hand, if the airline happens for reasons entirely its own to have particularly conscientious, professional employees, there is a potential for lowered chair repair costs.)

Electric chairs have a higher initial cost as well as larger maintenance expenses, so for airports with low traffic that get very little use out of their chairs, manual would be the way to go. Alternatively, in higher-traffic areas, the amount saved by employing fewer people to push around passengers in manual chairs will make up for the higher initial cost of power chairs. An electric chair will cost on average $1000 (Ebay, 2006), $500 more than a manual escort chair. There is an added maintenance cost, as well, with the electric chairs coming in at a median of $250 for annual upkeep as opposed to $125 for manual (Alexander, 1994-2005). Finally, with each use of the chair, the battery is depleted, and must from time be recharged or replaced by paid staff. Recharging certain batteries can take up to 10 hours (Rehabilitation Engineering). While this is not time actively spent, batteries under heavy use can require recharging every few days, even when used indoors (Wechsler, 2006). This would require extra batteries to be charging if we expect to get a full day's use out of a chair every day of the week. In reality, since the chairs are sitting unused for most of the non-peak hours of the day, assuming a battery change every three days, one spare battery per three chairs and a proper rotation should do the trick. The cost of the extra battery, which should last as long as a chair, is thus negligible. We can assume that, between unhooking a charged battery, taking it to the chair, changing batteries, and setting up the spent unit to charge, the time to change a single battery is 15 minutes, or about $4 for a technician making $20/hr. In bigger airports, where several batteries can be changed at once, this cost drops. Storage in a cold place, such as an outdoor shed on the tarmac, reduces battery life by about 50% for every 15 degrees Fahrenheit (8.3C) below 77F (25C), depending on battery type. Batteries are about $45 on the low end, but are rechargeable and commonly have year-long warranties (BatteryWholesale.com, 2001-2005).

Let's look first at a smallish airport that only has use for a few electric wheelchairs. We'll use the two-terminal Igor I Sikorsky Memorial Airport in Connecticut (Airnav.com, 2005) as an example, with 211 flights in the average day. We'll assume based on the airport statistics that the 53% of local traffic is largely done with single-engine planes, which generally carry 1-9 passengers and as such are neither required nor likely to have wheelchair compartments. Interregional traffic accounts for 42% of their operations, or about 89 flights daily on average. Assuming 100-passenger, midsize planes with wheelchair storage in the cabin, that's 8900 people a day, with a possibility of 1.2%, or just over 100 people, in wheelchairs, but not more than one or two on a flight on average. With 18 gates, it's unlikely even during peak hours more than 18 flights will arrive in the same hour. Of these, if a quarter are candidates for electric wheelchairs, an upper estimate on the number of electric wheelchairs considered by the airport would be 5. Since there are only two, small terminals, and in such a small airport there aren't many connecting flights involving planes that are handicap accessible, we'll say that the average time that

a person spends in transit is simply that required to go between the gate and baggage. The terminals are about 2400 feet long end-to-end (estimation based on the map provided on the referenced website), so at an average airport walking speed when obstacles are present of 125 ft./min (Young, 1999), it would take about 37 minutes to traverse the entire terminal, assuming the congestion persists. For a more sophisticated worst-case scenario, we can take the congestion to increase linearly as we near the parking lot, since the only people at the far end of a terminal are those whose flights deplaned there. For a corridor along which there are 7 gates, we'll put 1/7 of the traffic at the farthest gate from the parking lot, 2/7 of the traffic at the next gate in, etc, until the entire mass of humanity ends up at the last gate in their quest to exit the airport. We'll also make the assumption that walking speed decreases linearly with respect to congestion, with free-flow speed taken to be 263 ft/sec (Young, 1999). A linear fit yields:

(walking speed) = -138C+263 feet/minute

where C is a coefficient of congestion, 0 representing an uncongested walkway, 1 representing maximal normal congestion. (A linear fit makes sense as opposed to a discrete model because, while the people are coming from discrete pockets of population concentration, one's walking speed changes slowly in anticipation of a dense crowd. New users of electric chairs might have rather erratic driving models, but for lack of better information on the subject we'll assume their driving patterns model how they would walk, if a little jerkier.) In this model, $C = x/2400$, where x is the walker's position (in feet from the remote end) in the terminal. In order to find the time it takes to traverse the gate,

we need to solve the differential equation $\dfrac{dx}{dt} = -138\dfrac{x}{2400} + 263$. After some separation

of variables, an initial value calculation of t = 0, x = 0, and some rearranging of terms, we

get the equation $t = \left(\dfrac{-2400}{138}\right) \ln\left[-138\left(\dfrac{x}{2400}\right)+263\right] + 97$. For a position at the end

of the gate, x = 2400, we find t = 13 minutes. Note, if we start at the middle, as the mean traveler may do, the time till we get to the end of the terminal is 8 minutes. We'll allow a generous 6 minutes for getting people into and out of the chairs to avoid unduly hassling them. In the worst case, then, our passengers spend 19 minutes with our wheelchairs, though the median is likely to be closer to 14. This, in and of itself, isn't so wretchedly long, but taking a wheelchair "upstream" against a bunch of airline passengers trying to get home to their own beds can be quite the challenge. If it takes at least as long as pushing someone with the flow of traffic, the turnaround time for the chairs during busy hours can be reasonably estimated at 28 minutes in the average and 38 on the outside. Traffic could probably be accommodated with 4 chairs, two at each terminal.

     Assuming normal traffic, during which the chairs are be brought back to the gates as people use them, electric chairs would allow an employee to be freed up for, based on

previous estimates, roughly 15-20 minutes each time the chair is used. Employees are required to round up chairs, but that would be part of their job regardless. Assuming an hourly wage of $15, that's a savings of $3.75-$5 per trip. In order to pay off the $500 more the chair costs together with $625 in extra maintenance costs ($125 per year for five years), for now neglecting battery charging, in five years each chair would have to make 225-300 trips, or .12-.16 trips per day. At such low usage, not only would battery charging be a negligible cost, but the chairs would be likely to last longer than under normal wear and tear.

Price Comparison for Acquiring a Fleet of 5 Wheelchairs:
Assuming each chair is operated for an average of 2 hours a day year-round
(other costs, such as storage, apply uniformly between electric and manual)
All prices give in US Dollars

	5 Manual	5 electric	2 manual, 3 electric	
capital:				
5 chairs	2 500	5 000	4 000	
spare batteries	0	150	75	one per every three electric units
total startup:	2 500	5 150	4 075	
yearly costs:				
maintenance	625	1 250	1 000	
battery charging	0	1 825	1 095	every four days, $4 per chair
escorts	54 790	0	21 900	$15 per hour used per chair
total per year:	55 400	13 375	32 145	
total after five years:	279 500	72 025	164 800	

For a Fleet of 10 electric wheelchairs:

Capital:	10 000
Yearly:	26 750
5 years' total:	143 750

(Conclusion: A fleet of twice as many electric wheelchairs is still less expensive over 5 years than hiring escorts to push manuals.)

The above comparison shows that the use of electric wheelchairs, where persons are not independently mobile but do not require a chaperone, have the potential to be quite moneysaving. Below follows a preliminary analysis of a more general average case.

Single Concourse Configuration:

        Based on a walking distance distribution graph, which shows a basically linear trend from 0 to 425 meters distance walked (Wirasinghe), the median walking distance in a single-concourse terminal is 300 m, or 984 feet. (This figure includes hub transfers, normal transfers, arrivals, and departures.) Assuming a small amount of congestion, and thus average walking speed equivalent to that observed in walkers passing downstream obstructions observed in Young, 1999, of 204 ft/min, we'll see an average walking time of 4.8 minutes. This allows for a rapid turnaround time for the chairs, to the point that, in a pinch, one or several chairs could make two trips to pick up passengers from a single flight without forcing the immobile passengers to wait for more than the 30 unattended minutes allowable by law (ACAA).

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