1. (Stereographic coordinates)
Let M be the sphere $x^2 + y^2 + z^2 = 1$, and let $N = (0, 0, 1)$ be the north pole. Given a point p in the plane, we let q be the point on S on the line segment from N to p. For example, if $p = (0, 0, 0)$, then $q = (0, 0, -1)$.

a) Let $p = (u, v, 0)$. Find a formula for q in terms of u and v. Then find a formula for u and v given q.

b) Show that the map taking p to q is a conformal map.

2. Mercator's parameterization of the sphere is given by $\mathbf{x}(u, v) = (\operatorname{sech} u \cos v, \operatorname{sech} u \sin v, \tanh u)$. A chart into the cylinder is given by $\mathbf{y}(u, v) = (\cos v, \sin v, u)$. Show that the map taking $\mathbf{y}(u, v)$ to $\mathbf{x}(u, v)$ is a conformal map. Maple usage is encouraged.

3. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ and $g : \mathbb{R}^2 \to \mathbb{R}$ are smooth functions. Show that the map taking

$$(u, v, 0)$$

to

$$(f(u, v), g(u, v), 0)$$

is a local conformal map if and only if either

$$\begin{aligned} f_u &= g_v \\ f_v &= -g_u \end{aligned}$$

or

$$\begin{aligned} f_u &= -g_v \\ f_v &= g_u \end{aligned}$$

4. Suppose F and G are symmetric bilinear forms on the vector space V such that there exists $c \in \mathbb{R}$ such that $F(v, v) = cG(v, v)$ for all $v \in V$. Show that $F = cG$.

5. Oprea 3.1.6

6. Oprea 3.1.7

7. Oprea 3.3.4 (A line of curvature is a curve who's tangent vector is always a principal direction)

8. TBA