1. Consider the maps $\mathbf{x}(u,v) = (u,v,\sqrt{1-u^2-v^2})$ and $\mathbf{y}(a,b) = (a,\sqrt{1-a^2-b^2},b)$ which are charts into the sphere both having the domain (the open unit ball in $\mathbb{R}^2$).

We discussed in class the transition function of two charts is $\mathbf{y}^{-1}\circ\mathbf{x}$. Let $\tau = \mathbf{y}^{-1}\circ\mathbf{x}$, let V be its domain, let W be $\mathbf{x}(V)$, and let $Z = \tau(V)$.

For the two charts in this problem, explicitly compute what τ, V, W, and Z are. Verify that $\tau : V \to Z$ is a smooth function.

2. For each of the following surface patches, compute the equation of the tangent plane at the given point.

a) $\mathbf{x}(u,v) = (u,v,u^2-v^2)$, $(1,1,0)$

b) $\mathbf{x}(r,\theta) = (r\cosh\theta, r\sinh\theta, r^2)$, $(1,0,1)$

3. Consider the helicoid $\mathbf{x}(u,v) = (v\cos u, v\sin u, bu)$ where b is constant. Let $p = \mathbf{x}(u,v)$ be a point on the surface, and let U be the unit normal there.

Compute $U(u,v)$. Show that the cotangent of the angle between $U(u,v)$ and the z-axis is proportional to distance between p and the z-axis.

4. Oprea 2.2.5

5. Oprea 2.2.8

6. Oprea 2.2.15. Do this one without Maple.

7. Oprea 2.2.16. Feel free to recruit Maple on this one.