All parts of this homework to be completed in Maple should be done in a single worksheet. You can submit either the worksheet by email or a printout of it with your homework.

1. The point of this problem is to find a continuous function $f : \mathbb{R}^2 \to \mathbb{R}$ that has directional derivatives defined at $(0,0)$ but is not differentiable there.

a) Find a smooth 2π-periodic function $h(\theta)$ that has the following properties.

- $h(\theta) \geq 0$ for $0 \leq \theta \leq \pi$.
- $h(\theta) \leq 0$ for $\pi \leq \theta \leq 2\pi$.
- $h(\theta)$ vanishes exactly at the multiples of $\pi/2$
- $h'(0) > 0$.

Hint: First find a function $g(\theta)$ such that $g(\theta) \geq 0$ and $g(\theta)$ vanishes at multiples of $\pi/2$.

b) Find a function $f(x,y)$ satisfying

- $f(\cos(\theta), \sin(\theta)) = h(\theta)$
- $f(cx, cy) = cf(x,y)$ for any $c \in \mathbb{R}$.

c) Show that your function is continuous at $(0,0)$.

d) Compute the directional derivative of f at $(0,0)$ in the direction (a,b).

e) What are the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ at $(0,0)$? Don't work hard!

f) Explain why f cannot be differentiable at $(0,0)$.

g) Make a helpful plot in Maple of the function $f(x,y)$.

h) Using Maple or otherwise, compute $\partial f/\partial y$ along the x-axis and along the y-axis. Explain why this computation shows that $\partial f/\partial y$ is not continuous at $(0,0)$.

2. Suppose U is an open subset of $\mathbb{R}^n$, α is a differentiable curve in U, t_0 is in the domain of α, and $f : U \to \mathbb{R}$ is differentiable at $\alpha(t_0)$. Show that $f \circ \alpha$ is differentiable at t_0 and

$$(f \circ \alpha)'(t_0) = Df(\alpha(t)) \cdot \alpha'(t_0).$$

Hint:You know that $f(\alpha(t_0) + v) = f(\alpha(t_0)) + Df(t_0) \cdot v + R(v)$. Show that $|R(v)|$ can be written in the form $|R(v)| = S(v)|v|$ where $S(0) = 0$ and S is continuous at 0; to do this define $S(v) = |R(v)|/|v|$ for $v \neq 0$ and $S(0) = 0$.

3. Let $S = \{(x,y,z) : x^2 + y^2 = 1\}$. Find a single surface patch $(\mathbf{x}, U)$ such that $\mathbf{x}(U) = S$. You must verify that this map satisfies all properties of being a surface patch, including smoothness of the inverse.

4. Oprea 2.1.11 This exercise has minor mistakes in it. Part of your job is to find and correct them.

5. Oprea 2.1.20

6. Oprea 2.1.22. Make a plot in Maple that exhibits the surface as a (singly) ruled surface. Hint: look closely at the documentation for `plot3d` for instructions on how to plot a surface patch.