All parts of this homework to be completed in Maple should be done in a single worksheet. You can submit either the worksheet by email or a printout of it with your homework.

1. Oprea 1.4.10

2. Oprea 1.4.14 (Maple ok, but not required)

3. Oprea 1.5.2 (You'll have to read the paragraph just prior to understand the question.)

4. Oprea 1.5.3

5. Write a procedure in Maple that takes two arguments, a curve $\alpha(t)$ and the name of the parameter (t) and returns a list containing the following:

$$[v(t), \kappa(t), \tau(t), T(t), N(t), B(t)].$$

The procedure should attempt to create expressions that are as simple as possible. Demonstrate that your procedure works by applying it to the circular helix $\alpha(t) = (a\cos(t), a\sin(t), bt)$.

6. Use your procedure from the previous problem to assist you in answering Oprea 1.5.4 and 1.5.5.

7. In class we stated Green's theorem: if α is a smooth, simple, positively oriented, closed plane curve and $V = (P, Q)$ is a smooth vector field,

$$\int_{\text{int}(\alpha)} Q_x - P_y \, dxdy = \int_\alpha P\,dx + Q\,dy.$$

The theorem actually holds for piecewise C^1 curves, and the point of this exercise is to show that it holds when the domain is a polygon.

Demonstrate that the theorem holds in the case where α parameterizes a triangle. Then explain why Green's theorem holds for arbitrary polygons.

Here's a suggested approach for showing the result holds for triangles. If you can prove the result for vector fields that are always parallel to one side, or always perpendicular to one side of the triangle, you obtain the full result by linearity.

To prove the result for vector fields that are perpendicular to one side, you may assume (after applying a suitable coordinate transformation) that one vertex of the triangle is at the origin, the opposite side is parallel to the y-axis, and the vector field is parallel to the x-axis.

To prove the result for vector fields that are parallel to one side see if you can't reduce the problem to the previous case.