1. Let $\mathbf{x}$ be a chart. Define a family of charts $\mathbf{x}_\lambda(u,v)$ by

$$\mathbf{x}_\lambda(u,v) = \mathbf{x}(u,v) + \lambda U(u,v).$$

a) Let E_λ, F_λ, and G_λ be the coefficients of the first fundamental form of $\mathbf{x}_\lambda$. Define $E' = \frac{d}{d\lambda}|_{\lambda=0} E_\lambda$, and similarly for F' and G'. Show that

$$E' = -2l \qquad F' = -2m \qquad G' = -2n.$$

b) Prove that

$$\left.\frac{d}{d\lambda}\right|_{\lambda=0} (EG - F^2) = -4H(EG - F^2).$$

c) Let R be a region in the domain of $\mathbf{x}$ and let A_λ be its image under $\mathbf{x}_\lambda$. Show that

$$\left.\frac{d}{d\lambda}\right|_{\lambda=0} \text{Area}(A_\lambda) = \int_R -2H\sqrt{EG - F^2}\, du\, dv.$$

2. Let M be the surface of revolution obtained by revolving the curve $(f(t), g(t), 0)$ about the x-axis, where $a < t < b$. Show that the area of M is

$$2\pi \int_a^b g(t)\, dt.$$

Use this computation to compute the area of a torus with outer radius R and inner radius r.

3. Let M be a surface that is the image of a Monge patch $\mathbf{x}(u,v) = (u, v, f(u,v))$ where $(u,v) \in D$. Determine the area of M in terms of an integral over D involving f.

Use this computation to compute the area of the surface $z = x^2 + y^2$ where D is the unit disc.

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5. Write a Maple procedure that takes as an argument a chart $\mathbf{x}$, two parameter names (e.g. u v, an initial point (in (u,v) coordinates) and an initial velocity (u', v') and returns a procedure that gives a curve (in your choice of (u,v) or (x,y,z) coordinates) that represents the corresponding geodesic. You will want to use Maple's dsolve command with the numeric option here.

6. Use Maple to generate plots of geodesics on the one-sheeted hyperboloid $x^2 - z^2 = 1$ with angular velocities $\Omega = 0.5, 0.8, 1$ (both kinds), 1.2, and 2.

7. Use the Clairaut relation to give a qualitative description of all the geodesics on the torus. Be sure to discuss all cases of values of the angular momentum.