Proposition 2.33: Let A be a nonempty subset of $\mathbb{Z}$. Suppose for some $b \in \mathbb{Z}$ that $b \leq a$ for all $a \in A$. Then A has a least element.

Hint: Proving the Well-Ordering Principle was hard work. But proving this proposition should not be. Just reformulate it into a form where you can apply the Well-Ordering Principle.

Proposition HW7.A: Let $m, n, p \in \mathbb{Z}$ and suppose $p > 0$. If $mp \leq np$ then $m \leq n$.

Proof. Your proof here. □

Proposition 5.4: Let A, B, C be sets.

(i) $A = A$.

(ii) If $A = B$ then $B = A$.

(iii) If $A = B$ and $B = C$ then $A = C$.

Proof. Your proof goes here. Be lazy! □

Project 5.12 (partial): For each of the following double implications $P \iff Q$ determine which of the implications $P \implies Q$ or $Q \implies P$, if any, are true. For the ones that are true, prove them. For the ones are are not true, provide a counterexample.

(ii) $C \subseteq A$ or $C \subseteq B \iff C \subseteq (A \cup B)$

(iii) $C \subseteq A$ and $C \subseteq B \iff C \subseteq (A \cap B)$

Proposition 5.15 (DeMorgan's Laws): Given two subsets $A, B \subseteq X$,

(i) $(A \cap B)^c = A^c \cup B^c$

(ii) $(A \cup B)^c = A^c \cap B^c$

Proof. Your proof here. Be lazy! Look at Lemma 5.14a and Corollary 5.14b proved in class for useful results. □

Proposition 5.20: Let A, B, and C be sets.

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Proof. Your proof here. □