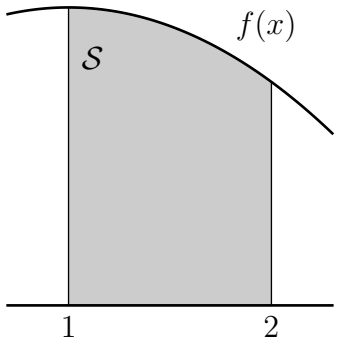
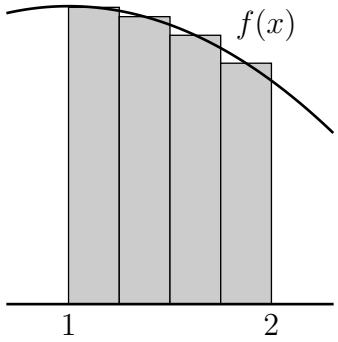
In this worksheet we will estimate the area of the region $\mathcal{S}$ between the x-axis and the graph of

$$f(x) = (x+1)(3-x)$$

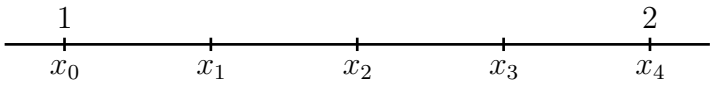
between $x = 1$ and $x = 2$.



Our strategy will be to estimate the area as a sum of areas of rectangles. To begin, we will use four rectangles as follows.



1. We break the interval $[1, 2]$ into four equally sized subintervals as follows.



What is the width Δx of each subinterval?

2. What are the values of x_0, x_1, x_2, x_3, and x_4?

3. Let $\hat{x}_k$ be the midpoint of interval k. What are the values of $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$ and $\hat{x}_4$?

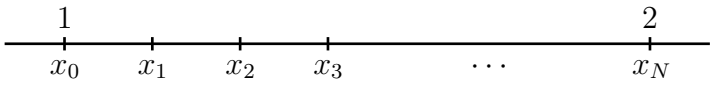
4. The height h_k of the k^{th} rectangle is the value of the $f(x)$ at the middle of the k^{th} interval. What are the heights h_1, h_2, h_3, and h_4?

5. The area A_k of the k^{th} rectangle is its width times its height. What are the four areas A_1, A_2, A_3 and A_4?

6. Estimate the area of the region $\mathcal{S}$ by adding A_1 through A_4.

7. We can get a better estimate for the area by using more rectangles. To do this, we will need to proceed systematically. Suppose we break the interval up into N subintervals; up until now N has been 4, but now we will not assume a particular value for N. We still break the interval $[1, 2]$ up into N rectangles of equal width Δx. How long is Δx?

8. We label the endpoints of the intervals as before:



Find a formula for x_k in terms of x_0, k, and Δx.

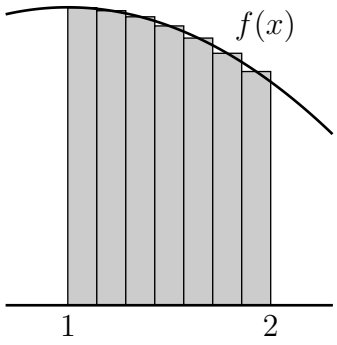
9. As before, let $\hat{x}_k$ be the midpoint of interval k. Find a formula for $\hat{x}_k$ in terms of the endpoints of interval k.

10. What is the height of the k^{th} rectangle? Express your answer in terms of the function $f(x)$ and your previous answer.

11. What is the area A_k of the k^{th} rectangle? Express your answer in terms of Δx and your previous answer.

12. Suppose you break the region up in to 37 rectangles. What is the area of the 11^{th} rectangle?

13. Suppose you use 7 rectangles to estimate the area.



What is the value of Δx?

14. Still assuming $N = 7$, complete the following table. Express your answers to 4 decimal places.

k	$\hat{x}_k$	h_k	A_k
1			
2			
3			
4			
5			
6			
7			

15. Estimate the area of $\mathcal{S}$ using 7 rectangles.