

1. For each of the matrices  $A$  below:

- Find the eigenvalues/eigenvectors.
- Determine the general solution of the differential equation

$$\mathbf{x}' = A\mathbf{x}$$

- Sketch, by hand, orbits of solutions in the phase plane. If your solution has the form  $\mathbf{x}(t) = c_1\mathbf{x}_1(t) + c_2\mathbf{x}_2(t)$ , you should include the orbits for  $\pm\mathbf{x}_1$  and  $\pm\mathbf{x}_2$ , as well as a few others.

a)

$$A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$$

b)

$$A = \begin{pmatrix} -1 & 1 \\ 2 & -2 \end{pmatrix}$$

c)

$$A = \begin{pmatrix} 5 & -3 \\ -2 & 10 \end{pmatrix}$$

d)

$$A = \begin{pmatrix} 0 & 9 \\ -1 & 0 \end{pmatrix}$$

2. Find the solution of

$$\mathbf{x}' = A\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

where

$$A = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix}.$$

When it comes time to solve for constants  $c_1$  and  $c_2$ , you must use Octave to solve for the constants, and you must indicate the Octave commands you used. Recall: to solve a matrix equation such as

$$\begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

the Octave commands are

```
octave-3.4.0:1> A=[3,4;1,3]
A =
```

3 4

1 3

```
octave-3.4.0:2> y=[1;2]
```

```
y =
```

1

2

```
octave-3.4.0:3> A \ y
```

```
ans =
```

-1

1

The solution is then  $x = -1$  and  $y = 1$ .