1. Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$$

a) Find the eigenvectors and eigenvalues of A.

b) Find two solutions of

$$\mathbf{x}' = A\mathbf{x}$$

of the form

$$\mathbf{x}_1(t) = e^{\lambda_1}\mathbf{v}_1; \qquad \mathbf{x}_2(t) = e^{\lambda_2}\mathbf{v}_2.$$

c) Find the general solution of

$$\mathbf{x}' = A\mathbf{x}.$$

d) Find the solution of $\mathbf{w}' = A\mathbf{w}$ satisfying

$$\mathbf{w}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

e) Use pp to make a phase plane for this system with $-3 \le x \le 3$ and $-3 \le y \le 3$. Then add, by hand, on top of your diagram, orbits of the following solutions: $\mathbf{x}_1$, $-\mathbf{x}_1$, $\mathbf{x}_2$, $-\mathbf{x}_2$, and $\mathbf{w}$. Label each of these orbits clearly.

2. Consider the differential equation

$$\mathbf{x}' = A\mathbf{x}$$

where

$$A = \begin{pmatrix} 6 & -3 \\ 2 & 1 \end{pmatrix}.$$

a) Find the eigenvectors and eigenvalues of A.

b) Write down the general solution of the differential equation.

c) By hand, sketch representative orbits in the plane. Your sketch must include any linear orbits, plus a few extra.