1. Section 5.2 Number 3. Set up the system, but **DO NOT SOLVE IT**.

2. Consider the IVP

$$x' = 3x - y$$
$$y' = 5x - 3$$

with $x(0) = 1$ and $y(0) = -1$. Use the method described in class to convert this system into a second order IVP for y. Solve this equation, and determine exact solutions $x(t)$ and $y(t)$.

Use the Octave function RK4 to find a numerical solution of this equation, and verify that the numerical solution and your exact solution are the same by plotting them together.

3. Consider the IVP

$$x' = 2x + y$$
$$y' = x + 2y - e^{2t}.$$

a) Convert this into a non-homogeneous second order equation for y.

b) Find the **general** solution y of this second order equation. Your solution should involve two free constants, c_1 and c_2.

c) Find the corresponding general solution x in terms of the same two free constants.

d) The general solution can be written in vector form as

$$\begin{pmatrix} x \\ y \end{pmatrix} = c_1\mathbf{v}_1(t) + c_2\mathbf{v}_2(t) + \mathbf{f}(t).$$

What are the vector-valued functions $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{f}$?

4. Consider the mass spring system

$$2y'' + y' + y = 0.$$

a) Convert this into a first order system for y and $v = y'$.

b) Use the Octave function pp to make a phase-plane with window $-3 < y < 3$ and $-3 < v < 3$.

c) By hand, on your plot, sketch the orbit of the solution starting at $y = 2$ $v = 0$.

d) Use the orbit that you just drew to then sketch, by hand, the solution curves $y(t)$ and $v(t)$.

Continued...

5. The orbits below correspond to three different solutions of a differential equation. For each of the three orbits, sketch the corresponding solutions $x(t)$ and $y(t)$ together on the same plot. (I.e. you are generating three graphs, each with two functions plotted on it.)

Note that the red orbit is traversed from bottom to top, the blue orbit is traversed from top to bottom, and the green orbit is traversed clockwise.