1. Consider the differential equation

$$y'' + 16y = 0.$$

a) Find the solution of this equation satisfying the initial conditions $y(0) = -2$ and $y'(0) = 12$. Write your solution in the form

$$y(t) = c_1 \cos(\beta t) + c_2 \sin(\beta t).$$

b) Rewrite this solution in the form

$$y(t) = A\cos(\beta t - \phi).$$

Determine the values of A and ϕ with $-\pi \le \phi \le \pi$.

c) Determine the phase shift, amplitude, period, angular and temporal frequencies, and time shift for this solution.

d) **By hand**, draw a sketch of $y(t)$. On your sketch, label the amplitude, the period, and the time shift.

2. Section 4.4 9–16. Report the general solution, not just a particular solution. Some basic rules for guessing a particular solution of

$$ay'' + by' + cy = f(t)$$

are as follows:

- If $f(t)$ is a polynomial, guess $y(t)$ is a polynomial of the same order.
- If $f(t) = e^{rt}$ guess $y(t) = Ce^{rt}$.
- If $f(t) = \sin(\omega t)$ or $f(t) = \cos(\omega t)$, guess $y(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t)$.
- If $f(t) = P(t)e^{rt}$ where $P(t)$ is a polynomial, guess $y(t) = Q(t)e^{rt}$ where $Q(t)$ is a polynomial of the same order as $P(t)$.
- If $f(t) = e^{rt}\cos(\omega t)$ or $f(t) = e^{rt}\sin(\omega t)$, guess $y(t) = e^{rt}(c_1 \cos(\omega t) + c_2 \sin(\omega t))$.
- If $f(t) = P(t)\cos(\omega t)$ or $f(t) = P(t)\sin(\omega t)$, guess $y(t) = Q(t)\cos(\omega t) + R(t)\sin(\omega t)$ where $Q(t)$ and $R(t)$ are polynomials of the same order as $P(t)$.

These rules work most of the time, but there are caveats we'll discuss on Monday. None of these problems involve the caveat.

3. Section 4.4 18, 20, 22. These ones are harder because they involve the caveat mentioned above; we'll talk about how you handle them on Monday.

4. Section 4.5 1. We'll talk about this kind of problem on Monday as well.