In class we proved the following. You can use it; **do not** reprove it.

Proposition 2.D: Let $m, n, p \in \mathbb{Z}$ and suppose $p > 0$. Then $m < n$ if and only if $mp < np$.

Now prove, while being as lazy as possible (i.e. taking advantage of the fact that we proved Propositions 2.D and 2.12) the following.

Proposition 2.E: Let $m, n, p \in \mathbb{Z}$ and suppose $p < 0$. Then $m < n$ if and only if $mp > np$.

Proof. Your proof here. □

Proposition 2.F: Let $m, n, p \in \mathbb{Z}$ and suppose $p > 0$. Then $m \leq n$ if and only if $mp \leq np$.

Proof. Your proof here. □

Proposition 2.G: Let $m, n, p \in \mathbb{Z}$. Then $m < n$ if and only if $m + p < n + p$.

Proof. Your proof here. □

Be lazy when proving DeMorgan's Laws. Look at Lemma 5.14a and Corollary 5.14b proved in class for useful results.

Proposition 5.15 (DeMorgan's Laws): Given two subsets $A, B \subseteq X$,

(i) $(A \cap B)^c = A^c \cup B^c$

(ii) $(A \cup B)^c = A^c \cap B^c$

Proof. Your proof here. □

Proposition 5.20: Let A, B, and C be sets.

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Proof. Your proof here. □

Project 5.21: Let A, B, C, and D be sets. For each of the following statements, determine if it is true or false. If it is true, prove it. If it is false, give a counterexample.

(i) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$.

(ii) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

Proof. Your proof here. □

Proposition 4.5: For all $k \in \mathbb{Z}_{\geq 0}$, $k! \in \mathbb{N}$.

Proof. Your proof here. □

Proposition 4.7 (i): For all $k \in \mathbb{N}$, $5^{2k} - 1$ is divisible by 24.

Proof. Your proof here. □