Math F655: Homework 4 Due October 1, 2010

1. Text 1.18

2. Let $I : S^2 \to S^2$ be an isometry.

a) Show that there is a unique map $\tilde{I} : \mathbb{R}P^2 \to \mathbb{R}P^2$ such that

$$\begin{array}{ccc} S^2 & \xrightarrow{I} & S^2 \\ {\scriptstyle \pi}\downarrow & & \downarrow{\scriptstyle \pi} \\ \mathbb{R}P^2 & \xrightarrow[\tilde{I}]{} & \mathbb{R}P^2. \end{array}$$

b) Show that $\tilde{I}$ is an isometry. Recall that $d_{\mathbb{R}P^2}([x],[y]) = \min(d_{S^2}(x,y), d_{S^2}(x,-y))$.

c) If $n(x) = -x$, show that $\widetilde{n \circ I} = \tilde{I}$. Conclude that every isometry on $\mathbb{R}P^2$ that descends from an isometry on S^2 has a fixed point.

3. Text 1.21 (2) (i.e. only the (infinite) Möbius strip.) If your proof has gaps, don't panic. But please point out where they are.