

1. For those who have not already finished problem 3 of the midterm, you may do so.
2. Show that the area of an asymptotic triangle in  $\mathcal{H}^2$  is  $\pi$ .
3. The goal of this problem is to compute an analogue of Theorem 4.11 for polygons in  $\mathcal{H}^2$ .
  - a) Show that if  $P$  is a triangulated polygon, then

$$2(T - E + V) = -T + 2V_I + V_B$$

where  $V_I$  is the number of vertices inside the polygon and  $V_B$  is the number of vertices on the boundary. Hint: Modify the formula  $3T = 2E$  and use the fact that  $V_B = E_B$  (i.e. the number of boundary edges and vertices is the same.).

- b) If  $A(P)$  is the area of the polygon, show that

$$A(P) = \pi T - 2\pi V + \sum_k \alpha_k$$

where the sum is over the boundary vertices and  $\alpha_k$  is the angle **outside the polygon** between the boundary edges meeting at boundary vertex  $v_k$ .

- c) Show that

$$-A(P) = -\pi T + 2\pi V_I + \pi V_B + \sum_k (\pi - \alpha_k).$$

- d) Rewrite the previous formula in terms of  $\chi(P)$ .
    - e) What is the value of  $\chi(P)$  anyways?
    - f) What is the meaning of the sum  $\sum_k (\pi - \alpha_k)$ ? Note that  $\alpha_k$  lives between 0 and  $2\pi$ .