1. Find a parameterization of the Klein bottle in $\mathbb{R}^4$ with no self-intersections. Use some form of technology to draw a projection of it into $\mathbb{R}^3$.

2. Consider the upper half plane model of hyperbolic space. Let $x = ai$ and $y = bi$ where $a, b \in \mathbb{R}$. We would like to show that any geodesic curve joining x to y is necessarily contained in the imaginary axis. Here are two ways to do it.

a) Let $\gamma : [0,1] \to \mathcal{H}^2$ be any curve starting at x and ending at y. Use the definition of a geodesic as an action minimizing curve to show that γ cannot be a geodesic unless its image is contained in the imaginary axis. *Hint:* Show explicitly that you can otherwise lower the action.

b) Give an argument using the strategy of Problem 4.7

3. Exercise 2.13 You are encouraged to take advantage of any and all results obtained in the course and on the homework.

4. In the final stage of the proof of the classification theorem, the presentation consisted of torus blocks and twisted adjacent pairs. We claimed that if there was a single twisted adjacent pair, then we could convert all torus blocks to twisted adjacent pairs. Show that this is true. That is, find cut-paste maneuvers to convert

$$Waba^{-1}b^{-1}cc^{-1}$$

to

$$Wdd^{-1}ee^{-1}ff^{-1}$$

where W is some arbitrary word not involving $a, \ldots, f$.

5. Exercise 2.9 Don't make this a long proof.

6. Calculate the hyperbolic radius, the hyperbolic center, and the hyperbolic area of the circle in $\mathcal{H}^2$: $|z - 2i - 1|^2 = 9/4$