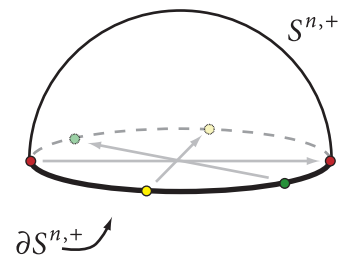


1. Let $\pi : X \rightarrow Y$ be a surjective continuous map. Show that π is a quotient map if and only if it takes saturated closed sets to saturated closed sets.
2. Let $\pi : X \rightarrow Y$ be a quotient map. Show that if A is a saturated open set or a saturated closed set, then $\pi|_A : A \rightarrow \pi(A)$ is a quotient map.
3. Let X be the line with two zeros. That is, X is the quotient space of $\{0, 1\} \times \mathbb{R}$ given by the equivalence relation $(0, x) \sim (1, x)$ if $x \neq 0$. Show that X is locally Euclidean and second countable, but not Hausdorff.
4. Suppose X is a second countable space. Let $\{U_\alpha\}_{\alpha \in I}$ be a collection of open sets in X such that $\cup_\alpha U_\alpha = X$. (We call $\{U_\alpha\}_{\alpha \in I}$ an *open cover* of X .) Prove that there exists a countable sub-collection $\{U_{\alpha_i}\}$ such that $\cup_i U_{\alpha_i} = X$. (We say that every open cover of a second countable space has a countable subcover).
5. Suppose π is a quotient map from a second countable space X to a locally Euclidean space Y . Prove that Y is second countable. *Hint:* Problem 4 might be handy.
6. (The Characteristic Property of the Quotient Topology is characteristic.) Suppose $\pi : X \rightarrow Y$ is a surjective function. Suppose that for any topological space Z that a function $f : Y \rightarrow Z$ is continuous if and only if $f \circ \pi$ is. Show that Y has the quotient topology induced by π .

$$\begin{array}{ccc}
 X & & \\
 \pi \downarrow & \searrow f \circ \pi & \\
 Y & \xrightarrow{f} & Z
 \end{array}$$

7. Show that the upper half sphere $S^{n,+}$ with antipodal points on $\partial S^{n,+}$ identified is homeomorphic to $\mathbb{R}P^n$.



Gluing a half sphere.