Note: This is a short homework! You will be receiving a take-home midterm on Wednesday, so you will want to get a head start on the current homework this weekend.

The following lemmas that we are in the midst of proving in class will be helpful on this homework.

Lemma 1. *Suppose* $f : [a, b] \to \mathbb{R}$ *is monotone increasing. If* $c \in (a, b]$*, then*

$$\lim_{x \to c^-} f(x)$$

exists and $\lim_{a \to c^-} f(c) \leq f(c)$*. If* $c \in [a, b)$*, then*

$$\lim_{x \to c^+} f(x)$$

exists and $\lim_{x \to c^+} f(x) \geq f(c)$.

Lemma 2. *Suppose* $f : [a, b] \to \mathbb{R}$ *is monotone increasing. If* f *is discontinuous at* $c \in (a, b)$*, then* f *has a jump discontinuity at* c*. If* f *is discontinuous at* a *or* b*, then it has a removable discontinuity there.*

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1. Suppose $f : A \to \mathbb{R}$, $c \in A$, and c is a limit point of both $A \cap (c, \infty)$ and $A \cap (-\infty, c)$. Prove that $\lim_{x \to c} f(x)$ exists and equals L if and only if $\lim_{x \to c^+} f(x)$ and $\lim_{x \to c^-} f(x)$ both exist and are equal to L.

2. 4.5.4 (*Hint:* What happens when a monotone increasing function is discontinuous?)

3. 4.5.6

4. 4.4.2

5. Suppose $f : A \to \mathbb{R}$ is uniformly continuous. Prove that f takes Cauchy sequences to Cauchy sequences. Also, find a function $f : (0, 1] \to \mathbb{R}$ that is continuous but does not take always take Cauchy sequences to Cauchy sequences.

6. 4.4.7