The integers modulo n, $\mathbb{Z}_n$, consist of the elements $0, 1, \ldots, n-1$. To add or multiply two numbers a and b, you add or multiply them as usual for integers, and then you find the remainder of your answer upon division by n. If n is prime, you can show that $\mathbb{Z}_n$ is an algebraic field.

The two smallest cases are $\mathbb{Z}_2$ and $\mathbb{Z}_3$. Here are the addition and multiplication tables:

+	0	1
0	0	1
1	1	0

×	0	1
0	0	0
1	0	1

+	0	1	2
0	0	1	2
1	1	2	3
2	2	0	1

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

1. Draw a picture of $\mathbb{Z}_2$ and a picture of $(\mathbb{Z}_2)^2$.

2. How many lines are there in $(\mathbb{Z}_2)^2$? Be careful you don't miss one!

3. Given a line in $(\mathbb{Z}_2)^2$, how many lines are parallel to it?

4. How many points at infinity should there be in the corresponding projective plane, $\mathbb{Z}_2\mathbb{P}^2$?

5. How many points are there in $\mathbb{Z}_2\mathbb{P}^2$?

6. Including the line at infinity, how many lines are there in $\mathbb{Z}_2\mathbb{P}^2$?

7. Try to draw a picture of $\mathbb{Z}_2\mathbb{P}^2$. When you are done, look at Figure 5.20 in your text. When you get a chance, you should try to find a correspondence between the picture you drew and the picture in the text.

8. Draw a picture of $(\mathbb{Z}_3)^2$.

9. A non-zero vector in $(\mathbb{Z}_3)^2$ is of the form (a, b) where $a \neq 0$ or $b \neq 0$. Explain why if $a \neq 0$, then there is an identical direction of the form $(1, b')$. What algebraic property of $\mathbb{Z}_3$ did you use in your answer?

10. How many different slopes can a line through the origin in $(\mathbb{Z}_3)^2$ have?

11. How many lines through the origin are there in $\mathbb{Z}_3\mathbb{P}^2$?

12. How many points at ∞ should there be in $\mathbb{Z}_3\mathbb{P}^2$? How many points are there in $\mathbb{Z}_3\mathbb{P}^2$?

13. Given a line in $(\mathbb{Z}_3)^2$, how many lines do you think are parallel to it? You do not need to prove your answer.

14. How many lines are there in $(\mathbb{Z}_3)^2$? How many lines are there in $\mathbb{Z}_3\mathbb{P}^2$?

15. Conjecture how many points and lines there are in $\mathbb{Z}_p\mathbb{P}^2$ where p is prime.