Assignment #1

REVISED: Due Monday 28 January, 2013 at the start of class

Please read Lectures 1, 2, 3 in the textbook *Numerical Linear Algebra* by Trefethen and Bau. A major purpose of this assignment is to familiarize you with MATLAB. You will need to find, download, or purchase a copy of MATLAB (or alternatives OCTAVE or PYTHON; see the "Comparison" handout). Make sure you can create a new program (text file), save it, edit it, and run it at the command line by typing its name.

Do these exercises:

P1. Consider the 3×3 real matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ -2 & 0 & 1 \\ 4 & 4 & 6 \end{bmatrix}$$

By hand and on paper:

- compute the rank and determinant of A,
- compute the eigenvalues of A, and
- compute the inverse of the 2×2 matrix $B =$ `A(1:2,1:2)`.

Now check your work at the MATLAB command line. (*You'll use these* MATLAB *commands; type* "`help det`" *if needed*: `rank, det, eig, inv`.)

P2. (*This problem is about the most basic algorithms of linear algebra, and about their elementary implementations. Nothing fancy. You'll need to be careful with indices!*)

(a) On paper, write down a 4×3 matrix A and a 3×1 column vector v with integer entries of your choice, and compute the product Av by hand. Again on paper, write down another 3×3 matrix B and compute AB by hand. Now check your work at the MATLAB command line by entering these matrices into MATLAB and using "$*$".

(b) On paper write the algorithm which computes the product of a rectangular matrix $A \in \mathbb{C}^{m \times n}$ and a column vector $v \in \mathbb{C}^{n \times 1}$. Use the textbook's notation for vector and matrix entries. Count the number of floating point operations exactly (i.e. as an exact expression in m and n). Now implement this algorithm in a MATLAB program `matvec.m` which is a function; the first line will say

```
function z = matvec(A,v)
```

Please extract the sizes of the input objects using the MATLAB command `size`. Use `error` if these user-given objects are the wrong sizes. Use `for` loops. (*Yes, I know it can be done by colon notation. I want you to think about the underlying implementation.*) Check your code on the example you wrote down in part **(a)**.

(c) Do the same steps for the product of matrices $A \in \mathbb{C}^{m \times n}$ and $B \in \mathbb{C}^{n \times k}$. Call your function "`matmat`" and check your code on the example you wrote down in part **(a)**.

P3. (*This problem will also familiarize you with* MATLAB. *Use these additional* MATLAB *commands*: `randn, norm, abs, mean, plot, loglog`.)

Write a MATLAB script (i.e. a program which is not a function) to generate 10 random matrices of size $m \times m$ for each of these powers of two: $m = 2, 4, 8, \ldots, 512$. The matrix entries should be normally-distributed random real numbers with mean zero and standard deviation one. For each of these matrices compute the rank, the 2-norm, the 2-norm of the inverse, and the absolute value of the determinant. Communicate these data in reasonable ways.

Because I am proposing you generate 90 matrices so I absolutely do not want a giant table of the values of their determinants and so on, much less the matrices themselves! Instead, use sentences to state conclusions, and use plots to communicate data and patterns. Computes averages over the 10 tries as needed, but note that sometimes the raw data can appear nicely in a plot.

Exercise 1.1 in Lecture 1.

Exercise 1.3 in Lecture 1.