Final Exam: Recent Part

100 points total for this part. You have 120 minutes for both parts together.

1. (*10 pts*) Find the two real numbers whose difference is 18 and whose product is a minimum.

2. (*10 pts*) Find $f(x)$ for which $f''(x) = 3x^2 - 2$ and $f(2) = 0$ and $f'(2) = 1$.

3. **(a)** (*8 pts*) Write the integral as the limit of a sum, using right end points for the locations where the function (integrand) is evaluated:

$$\int_0^2 3x - 4\,dx$$

In particular, state formulas for Δx and x_i.

(b) (*7 pts*) Compute the limit in part **(a)**, using the fact that $\sum_{i=1}^n i = \frac{1}{2}n(n+1)$.

(c) (*7 pts*) To check your work in the above parts, compute using the Fundamental Theorem of Calculus (i.e. FTC II):

$$\int_0^2 3x - 4\,dx$$

4. (*10 pts*) Compute

$$\int x^2 \sec^2(x^3+1)\,dx$$

5. (*10 pts*) Compute $f'(x)$ if

$$f(x) = \int_{x^2}^3 \cos(e^t)\,dt$$

6. Consider the curve

$$y = \frac{x^2 - 4}{x^2 - 1}.$$

(a) (*5 pts*) Is this curve even, odd, or neither?

(b) (*5 pts*) Find all vertical and horizontal asymptotes of the curve.

(c) (*5 pts*) Find the critical numbers.

(d) (*5 pts*) Sketch the graph.

7. (*8 pts*) Compute the limit:

$$\lim_{x \to 0} \frac{\tan x - x}{1 - \cos x}$$

8. (*10 pts*) Find the volume of the solid obtained by rotating the region bounded by the curves

$$y = x(1 - 1), \qquad y = 0$$

about the x-axis. Sketch the region, the solid, and a typical disc or washer.

Extra Credit. (*3 pts*) Compute

$$\int \frac{x^3}{\sqrt{1 - x^2}}\, dx$$