## Final Exam: Comprehensive Part

100 points total for this part. You have 120 minutes for both parts together.

- **1.** (a) (5 pts) Compute f'(5) if  $f(x) = x^{-2}$ .
- (b) (5 pts) Use the definition of the derivative to compute f'(5) if  $f(x) = x^{-2}$ .
- 2. (a) (5 pts) Find dy/dx if  $y = 2\sqrt{x} + 6e^{x}$

(b) (5 pts) Compute

$$\int 2\sqrt{x} + 6e^x \, dx$$

**3**.  $(10 \ pts)$  The equation

$$x^2 - xy + y^2 = 4$$

is an ellipse. (2,0) is a point on this ellipse. Find the equation of the tangent line to this ellipse at this point. (Hint: *There is no need to sketch any graph.*)

4. Consider the curve

$$y = \frac{4-x}{x-1}.$$

(a) (5 pts) Is this curve even, odd, or neither?

(b) (5 pts) Find the intercepts (locations where the graph crosses the x and the y axes).

(c) (5 pts) Find the intervals on which the curve is increasing and decreasing.

(d) (5 pts) Sketch the graph, showing any asymptotes.

5. (10 pts) Find the absolute maximum and minimum of  $f(t) = 2t - \tan t$  on the interval  $[0, \pi/4]$ .

## Final Exam: Comprehensive Part **CONTINUED**

SAMPLE EXAM

6. (a)  $(5 \ pts)$  Bismuth-210 is a radioactive substance whose mass decays exponentially. It has a half-life of 5.0 days. A sample originally has a mass of 800 mg. Find a formula for the mass y(t) remaining after t days.

(b) (5 pts) Find the mass remaining after 15 days.

**7**. 
$$(10 \ pts)$$
 Define

$$\lim_{x \to a} f(x) = L "$$

You may use either the sentence definition written many times in lecture, or you may use the  $\epsilon - \delta$  definition.

8. (10 pts) Compute

$$\int_3^7 \frac{\cos x}{\sin^4 x} \, dx$$

**9**. (10 pts) Find the area between  $y = e^x$  and y = 1/x on the interval [1, 2].

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**Extra Credit**. (3 pts) Show that

$$\frac{d}{dx}\left(\operatorname{arcsec} x\right) = \frac{1}{x\sqrt{x^2 - 1}}$$

You may use known derivatives of trignometric functions, but not, naturally, the derivatives of inverse trigonometric functions. You may use standard trigonometric identities.