NAME: ______________________

MATH 200 Calculus 1 (Bueler) 12 November, 2008

Midterm Exam # 2

100 points total. You have 60 minutes.

1. Find the derivatives.

(a) (*5 pts*)

$f(x) = \sec x + \ln x$

(b) (*5 pts*) *Find dy/dx.*

$$y = \frac{x^2 + x}{x - 4}$$

(c) (*5 pts*) *Find dy/dx.*

$e^y \sin x = 1 + \cos y$

(d) (*5 pts*)

$g(s) = \log_2(1 - 3s)$

2. **(a)** (*5 pts*) Using the known derivatives of $\sin x$ and $\cos x$, show that

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

(b) (*10 pts*) Show that

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

Hint: Use implicit differentiation. And you will need to use a trigonometric identity to simplify the result.

3. (*10 pts*) Find the equation of the line tangent to the curve:

$$y = \sin(\sin(x)), \qquad (\pi, 0)$$

4. **(a)** (*5 pts*) Find the critical numbers of the function $s(t) = t^4 - 2t^2 + 2$.

(b) (*5 pts*) Find the locations and values of the absolute maximum and minimum of the same function, $s(t) = t^4 - 2t^2 + 2$, on the interval $[-1, 2]$. *You may use the information from* **(a)**.

5. (*10 pts*) If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 8 cm.

6. The half-life of cesium-137 is 30 years. Suppose we have a 1000-mg sample.

(a) (*5 pts*) Find the mass that remains after t years.

(b) (*5 pts*) After how long would will only 1 mg remain? (*There is no need to simplify your expression as long as it is correct.*)

7. (*10 pts*) Sketch the graph of a function f that is continuous on the interval $[1, 5]$, has no local minimum or local maximum, but for which 2 and 4 are critical numbers.

8. **(a)** (*5 pts*) Compute the linearization (linear approximation) $L(x)$ of the function $f(x) = e^x$ at the point $a = 0$.

(b) (*5 pts*) Use the linearization to approximate $e^{0.03}$.

(c) (*5 pts*) Graph the function $y = f(x)$ and its linearization $y = L(x)$ on the same axes. *Give a scale on each axis, and label which graph is which.*

Extra Credit. (*3 pts*) Show using Rolle's theorem that a polynomial of degree 3 has at most three real roots.