NAME: _____

MATH 200 Calculus 1 (Bueler) 8 October, 2008

Midterm Exam # 1

100 points total. You have 60 minutes.

1. **(a)** (*5 pts*) On the axes provided, sketch a graph of $y = 2^x$. (*Be sure to give some scale to each axis, for example by identifying coordinates of some points on the graph.*)

(b) (*5 pts*) If $f(x) = 2^x$ and $g(x) = \sqrt{x-1}$, give the formula for, and the domain of, $(g \circ f)(x)$.

2. (*5 pts*) Find the derivative of $F(r) = r^3 + e^r$.

3. Compute the limits:

(a) (*5 pts*)

$$\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$$

(b) (*5 pts*)

$$\lim_{x\to \infty} \frac{x+2}{\sqrt{9x^2+1}}$$

4. (*10 pts*) On the axes provided, sketch a graph of a function with these properties:

$$\lim_{x\to -1^-} f(x) = 0, \qquad \lim_{x\to -1^-} f(x) = -1, \qquad f(0) = 0, \qquad f(1) = 3, \qquad f'(1) = -3.$$

5. (*10 pts*) Use the definition of the derivative to find $f'(x)$ if $f(x) = x^2 + 2$.

6. (*10 pts*) Find all the vertical and horizontal asymptotes of the graph

$$y = \frac{x^3 + 8}{x^2 - x}$$

7. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t - 1.86t^2$.

(a) (*5 pts*) Compute the average velocity over the interval $[1, 2]$. (*There is no need to simplify the number, but give a formula which would be easy to evaluate on a calculator.*)

(b) (*5 pts*) Set up and then compute a limit to calculate the instantaneous velocity at $t = 1$.

(c) (*5 pts*) Compute dy/dt if $y = 10t - 1.86t^2$.

8. (*10 pts*) Define the statement

$$\text{“ } \lim_{x \to a} f(x) = L. \text{ ”}$$

Give either the complete sentence definition or the “ϵ, δ” definition.

9. **(a)** (*5 pts*) Use the definition of continuity and the properties of limits to show that the function $f(x) = \cos 5x$ is continuous at $x = 0$.

(b) (*5 pts*) Compute the limit:

$$\lim_{x \to 0} \left(x^3 + \frac{\cos 5x}{10000} \right)$$

10. (*10 pts*) Use the given graph of $f(x) = 1/x$ to find a number δ such that

$$\text{if} \qquad |x-2| < \delta \qquad \text{then} \qquad \left|\frac{1}{x} - 0.5\right| < 0.2$$

graph in 2.4 # 1

Extra Credit. (*3 pts*) Show that the equation

$$x^4 + x - 3 = 0$$

has at least two solutions, and state the facts and name the theorems you use.