Study Guide for Midterm Exam I

The Midterm Exam is in-class on Wednesday 20 February, 2013.
The exam is closed-book and no calculators are allowed.

Review of the *ideas* in the sections:

1.1 A function satisfies the vertical line test. Graph functions by: "plugging-in" numbers, finding even/odd symmetry, finding x- or y-intercepts.

1.2 Know standard forms of lines, esp. point-slope and slope-intercept. Find the slope of the line from two points. The slope of parallel lines are equal. Perpendicular lines have negative inverse slopes ($m_1 = -1/m_2$).

1.3 For functions, what do "domain" and "range" mean? Find domain and range of specific functions. How do transformations change the graph of a function? What does the "composition" of two functions mean?

1.5 Inverse functions undo the function: $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. An inverse function can only exist if the original function is one-to-one (horizontal line test). Graph the inverse of a function. Inverse trig functions require choice of sub-domain of original trig function. Know graphs of inverse trig functions.

1.6 Rules (properties) for exponential functions. Define natural logarithm as inverse function of e^x. Properties of logarithms. Graph $y = e^x$ and $y = \ln x$.

2.1 Page 63 is a one page view of the goal of learning limits and derivatives.

2.2 My informal definition was: "We say

$$\lim_{x \to c} f(x) = L$$

if we can make the values of $f(x)$ as close as one wishes to L by choosing x-values sufficiently close to, but not equal to, c." Know both this definition and the formal definition of the limit on page 72. Compute limits by using table of values or graph.

2.3 The limit rules on page 79 imply that limits are easy to compute unless there is a zero in the denominator of a fraction. (*This is the most important "hard" case, anyway.*) This means there are lots of "easy" cases where you just compute the limit by plugging in the value. In hard cases, factor and cancel in fractions, or rationalize fractions. We did not cover the "squeeze theorem". Know special limits

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1, \qquad \lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} = 0.$$

2.4 A function is continuous if the limit is "easy" as above: you compute the limit by plugging in the value. Know what a "removable" discontinuity is. Know the meaning of one-sided limits. Intermediate Value Theorem: continuous functions on $[a, b]$ do not "jump past" any y-values between $f(a)$ and $f(b)$.

2.5 A function has an infinite two- or one-sided limit $\lim_{x \to c} f(x) = \pm\infty$ or $\lim_{x \to c^\pm} f(x) = \pm\infty$ if the values of $f(x)$ can be made as large ($+\infty$) or as small ($-\infty$) as desired by choosing x-values sufficiently close to, but not equal to, c. The graph $y = f(x)$ has a vertical asymptote $x = c$ if it has an infinite limit there. In practice, to decide on whether there is an infinite limit, and if it is $+\infty$ or $-\infty$, one should think about the values of the function for concrete x-values close to c.

3.1 Be able to find the slope of the secant line. The tangent line slope is the limit as the x-values c and $c+\Delta x$ come together:

$$m_{sec} = \frac{f(c+\Delta x) - f(c)}{\Delta x} \quad \text{becomes} \quad m_{tan} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}.$$

The derivative is the same limit as m_{tan}, but thought of as a new function:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}.$$

Yes, you are responsible for knowing this definition. Yes, you must be able to set up and calculate this kind of limit. *When I want you to calculate the derivative by using this limit definition, like you did for exercises in 3.1, I'll say* "Find the derivative by the limit process" *or* "Find the tangent line by the limit process".

3.2 With these rules you can calculate derivatives of polynomials and $\sin x, \cos x, e^x$. You should already be able to use them almost without thinking. Know rates of change idea for position, velocity, acceleration: $v(t) = s'(t)$ and $a(t) = v'(t)$.

3.3 Product and quotient rules. Know how to do examples and simplify answers. Know how to derive the rules for $\tan x, \cot x, \sec x, \csc x$ *and* memorize the resulting formulas. Know notation for second and higher derivatives:

$$f''(x) = y'' = \frac{d^2y}{dx^2} = \frac{d^2}{dx^2}\left[f(x)\right].$$

3.4 The chain rule is the key to differentiating any combination of functions:

$$\Big[f\left(g(x)\right)\Big]' = f'\left(g(x)\right) g'(x) \qquad \text{or} \qquad \frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}.$$

Practice looking at complicated functions and seeing them as an outer function applied to an inner function (…and then a next-inner function and so on). The "Basic Differentiation Rules" table in the inside front (or back) cover of your book states all the rules using the chain rule, such as

$$\frac{d}{dx}\left[\sin u\right] = \cos u\, u' \qquad \text{or} \qquad \frac{d}{dx}\left[u^n\right] = nu^{n-1}\, u'.$$

Finally, know the derivative rules for *all* exponentials and logarithms:

$$[e^x]' = e^x, \qquad [\ln x]' = \frac{1}{x}, \qquad [a^x]' = (\ln a)\, a^x, \qquad [\log_a x]' = \frac{1}{(\ln a)x}.$$

Remember to:

… read the question. Are you doing the operation which answers the question?
Don't do this: *The question says "Find the tangent line …" for some function. You just take the derivative of the function and stop there.*

… use "=" like you mean it. Are the objects on either side of "=" actually equal?
Don't do this: *In order to take the derivative of $f(x)$ you decide to rewrite it. But then you end up saying that $f(x) = f'(x)$, which is nonsense.*

… do lots of exercises and read lots of examples as practice for the exam.