SAMPLE Final Exam

RECENT Material

1. Given $\int_0^5 f(x)\,dx = 10$ and $\int_0^7 f(x) \ = 6$, evaluate $\int_5^7 f(x)\,dx$.

2. Solve the differential equation problem: $f''(x) = 3x^2 - 2$ and $f(2) = 0$ and $f'(2) = 1$.

3. **(a)** Write the integral as the limit of a sum, using equally-spaced subintervals and right end-points:

$$\int_0^2 3x + 4\,dx$$

(Please state formulas for Δx and x_i as part of your solution.)

(b) Simplify the sum and compute the limit in part **(a)**. That is, use a summation formula and the limit process to find the area under the curve.

(c) To check your work in the above parts, compute the integral in part **(a)** using the Fundamental Theorem of Calculus.

4. Compute/evaluate the integrals:

(a)

$$\int_0^1 x^2 \sec^2(x^3 + 1)\,dx$$

(b)

$$\int \tan(5\theta)\,d\theta$$

(c)

$$\int \frac{\cos x}{\sin^4 x}\,dx$$

5. Compute $f'(x)$ if

$$f(x) = \int_3^{x^2} \cos(e^t)\,dt$$

6. Use the substitution $u = \sqrt{x}$ to evaluate

$$\int \frac{dx}{\sqrt{x}\,(1+x)}\,dx$$

7. Use $n = 4$ Simpson's rule to approximate

$$\int_1^5 \ln x\,dx.$$

(As you do not have a calculator, you may leave a concrete answer unsimplified.)

Extra Credit. Compute

$$\int \frac{x^3}{\sqrt{1-x^2}}\,dx$$

COMPREHENSIVE Material

8. Use calculus to find the two positive numbers whose sum is 18 and whose product is a maximum.

9. Consider the curve

$$y = \frac{x^2 - 4}{x^2 - 1}.$$

(a) Find all vertical and horizontal asymptotes of the curve.

(b) Find the open intervals on which the graph is increasing and decreasing.

(c) Sketch the graph.

10. **(a)** Compute $f'(5)$ if $f(x) = x^{-2}$.

(b) Use the limit definition of the derivative to compute $f'(5)$ if $f(x) = x^{-2}$.

11. **(a)** Find dy/dx if

$$y = 2\sqrt{x} + 6e^x$$

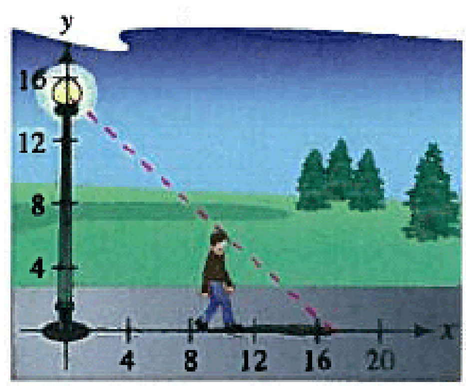
(b) Find the indefinite integral:

$$\int 2\sqrt{x} + 6e^x \, dx$$

12. The point $(2, 0)$ is on the graph $x^2 - xy + y^3 = 4$. Find the equation of the tangent line to this graph at this point. (Hint: *There is no need to sketch any graph.*)

13.

A man 6 feet tall walks at a rate of 3 feet per second away from a light that is 15 feet above the ground (see figure). When he is 10 feet from the base of the light, at what rate is the length of his shadow changing?



14. Define

$$\lim_{x \to c} f(x) = L$$

You may use either the sentence definition written many times in lecture, *or* you may use the ϵ–δ definition.

15. Show that

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1 - x^2}}$$

You may use known derivatives of trignometric functions, but not, naturally, the derivatives of inverse trigonometric functions. You may use standard trigonometric identities.